

An equation that involves one or more trigonometric ratios of a variable is a trigonometric equation.

ex. $\sin \theta = 0.5$

ex. $4 \cos x + 1 = 0$

ex. $2 \tan 2x - 5 \tan x - 3 = 0$

To solve linear trigonometric equations:

- 1) Isolate for $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, or $\cot\theta$.
- 2) Switch any reciprocal trig ratios to their corresponding primary trig ratio.
- 3) Use the inverse function on your calculator or special triangles and the CAST rule to find θ .

Examples: Solve the following equations in the interval $x \in [-2\pi, 2\pi]$

a) Find the exact values of x , for $\sin x = -1/\sqrt{2}$

b) Round answers to 3 decimal places, for $\tan x - 3 = 0$

c) Round answers to 3 decimal places, for $2 \sec x + 5 = 0$

To solve quadratic trigonometric equations:

- 1) Set one side equal to zero.
- 2) Let $\mathbf{a} = \sin x$ or $\cos x$ or $\tan x$ or $\csc x$ or $\sec x$ or $\cot x$. Then replace the trig functions with \mathbf{a} in the equation.
- 3) Factor the equation if possible. Then set each factor equal to zero and solve for \mathbf{a} .
- 4) If it is not possible to factor, use the quadratic formula to solve for \mathbf{a} .
- 5) Replace each \mathbf{a} with the appropriate trig function.
- 6) Solve each factor using your rules for solving linear trigonometric equations.

Examples: Solve each equation in the interval $x \in [-2\pi, 2\pi]$

a) $\cos 2x - 1 = 0$

b) $2 \csc 2x - \csc x - 1 = 0$

c) $5 \cot 2x - 2 \cot x - 3 = 0$

Example: The range of an arrow shot from a particular box can be modeled by the equation $r = 100\sin 2\theta$, where r is the range in metres and θ is the angle in radians above the horizontal that the arrow is released. A target is placed 80 m away.

- a) What are the restrictions on the angle θ ?
- b) Determine the angle or angles that the archer should use to hit the target, to the nearest hundredth of a radian.