Section 5.3 Transformations of Sinusoidal Functions

The transformation of a sine or cosine function f(x) to g(x) has the general form:

g(x) = a f [k (x - d)] + c

where $|\mathbf{a}|$ is the amplitude, if a < 0, there is a reflection in the x-axis **k** is the horizontal stretch or compression, if k < 0 there is a reflection in the y-axis **d** is the phase shift **c** is the vertical displacement.

The period is given by $2\pi / k$

Examples: Transform the function $f(x) = \sin x$ to g(x) such that $g(x) = 3 \sin \left[\frac{3}{2}x + \frac{\pi}{4}\right] - 2$ State the min and max values, the amplitude, the period, the phase shift the vertical displacement, and the domain and range. Graph both functions, f(x) and g(x), over 2 cycles.

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				4.0					
				4.0					
				3.0					
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				-2.0					
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_2π	-3π/2	π	-π	-1.0	π.	/2 1	τ 3π	./2 2	π
_2π	-3π/2	_π	-7		π.	/2 7	τ 3π	2 2	π
_2π	<u>-3</u> π/2	π		-1.0	π.	/2 7	τ 3π	2 2	π
_2π	<u>-3</u> π/2	π		-1.0 -2.0 -3.0	π.	⁽² 7	τ 3π	2 2	π
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_2π	3π/2	π		-1.0 -2.0 -3.0	π.		τ 3 <i>π</i>	2 2	π

Transforming Sinusoidal functions to Match Data not Given in Terms of π

*Remember that a period, p, is $p = 2\pi / k$ and that $k = 2\pi / p$.

Transform the function $f(x) = \cos x$ to g(x) such that g(x) has an amplitude of 2, a period of 1, a phase shift of 0.5 to the left and a vertical displacement of 3 units up. Graph the function over 2 cycles. Write the equation of the function.

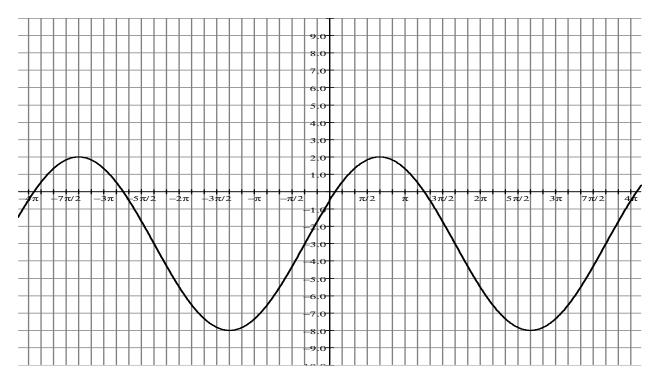
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	2.0	-1.0		0 2	0 3
	2.0	-1.0 -2.0 -3.0		0 2	0 3
	2.0 -:	-1.0 -2.0 -3.0		0 2	0 3
	2.0	-1.0 -2.0 -3.0		0 2	0 3
	2.0	-1.0 -2.0 -3.0 -4.0		0 2	0 3
		-1.0 -2.0 -3.0		0 2	0 3
	2.0	-1.0 -2.0 -3.0 -4.0		0 2	0 3
	2.0	-1.0 -2.0 -3.0 -4.0			0 3
	2.0	-1.0 -2.0 -3.0 -4.0		0 2	0 3

Example: Write an equation to represent the following functions.

a) A sine function with a maximum value of 5, a minimum value of -3, a phase shift of $5\pi/6$ rad to the right and a period of $2\pi/3$.

b) A cosine function has a maximum value of -2 and a minimum value of -3, a phase shift of 3 rad to the left and a period of 5.

Example: Given the graph below, write an equation using both a cosine and sine function.



Example: The vertical position, h, in metres, of a rider on a Ferris wheel, after time, t, in seconds, is a sinusoidal function. The maximum height above the ground is 22m and the minimum height is 2m. The Ferris wheel completes one turn in 30 seconds, and the model predicts the highest point at t = 0 seconds. Determine an equation to model the Ferris wheel's rotation as both a cosine and a sine function.