Section 4.5 Proving Trigonometric Identities

In this section, you will use the following basic trigonometric identities to prove other identities.

The Pythagorean Trig Identity (PTI): $\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$

Reciprocal Trig Identities (RTI): $\csc\theta = \frac{1}{sin\theta}$, $\sec\theta = \frac{1}{cos\theta}$, $\cot\theta = \frac{1}{tan\theta}$

Quotient Trig Identities (QTI): $\tan\theta = \frac{\sin\theta}{cos\theta}$, $\cot\theta = \frac{\cos\theta}{sin\theta}$

Compound Angle Formulas:

$\sin(a+b) = \sin a\cos b + \cos a\sin b$
$\sin(a-b) = \sin a\cos b - \cos a\sin b$
$\cos(a+b) = \cos a\cos b - \sin a\sin b$
$\cos(a-b) = \cos a\cos b + \sin a\sin b$

Double Angle Formulas:

$sin2\theta = 2sin\theta cos\theta$
$cos2\theta = cos^2\theta - sin^2\theta$
$cos2\theta = 2cos^2\theta - 1$
$cos2\theta = 1 - 2sin^2\theta$

Example: Prove $cos\left(\frac{\pi}{2} - x\right) = \sin x$

The goal is to show that the Left Hand Side (LHS) = Right Hand Side (RHS)

LHS:	RHS:
$cos\left(\frac{\pi}{2} - x\right)$	$\sin x$
$\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$	$\sin x$
$0(\cos x) + 1(\sin x)$	$\sin x$
$\sin x$	$\sin x$

Therefore LHS = RHS

Example: Prove $\csc 2\theta = \frac{\csc\theta}{2\cos\theta}$

RHS:
$$\frac{\csc\theta}{2\cos\theta} = \csc\theta\left(\frac{1}{2\cos\theta}\right)$$
$$= \frac{1}{\sin\theta}\left(\frac{1}{2\cos\theta}\right)$$
$$= \frac{1}{2\sin\theta\cos\theta}$$
$$= \frac{1}{\sin 2\theta}$$
$$= \csc 2\theta$$
$$= LHS$$

Therefore LHS = RHS

Example: Prove that $\cos(x+y)\cos(x-y) = \cos^2 x + \cos^2 y - 1$

Example: Prove that $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} - \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = 2\tan 2\theta$