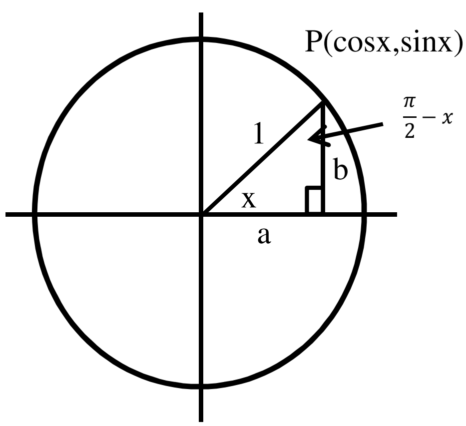
Section 4.3 Equivalent Trigonometric Expressions

- Equivalent trigonometric expressions are expressions that yield the same value for all values of the variable.
- An identity is an equation that is true for all values of the variable for which the expressions on both sides of the equation are defined.
- An identity involving trigonometric expressions is called a trigonometric identity.

Cofunction Identities

Using a Right Triangle and the Unit Circle to determine equivalent trigonometric identities featuring $\frac{\pi}{2}$.



$\sin x = b$	$\sin\left(\frac{\pi}{2}-x\right) = a$
$\cos x = a$	$\cos\left(\frac{\pi}{2}-x\right) = b$
$\tan x = \frac{b}{a}$	$\tan\left(\frac{\pi}{2}-x\right) = \frac{a}{b}$
$\csc x = \frac{1}{b}$	$\csc\left(\frac{\pi}{2}-x\right) = \frac{1}{a}$
$\sec x = \frac{1}{a}$	$\sec\left(\frac{\pi}{2}-x\right) = \frac{1}{b}$
$\cot x = \frac{a}{b}$	$\cot\left(\frac{\pi}{2}-x\right) = \frac{b}{a}$

So in Quadrant 1

$$\sin\left(\frac{\pi}{2}-x\right) = \cos x$$

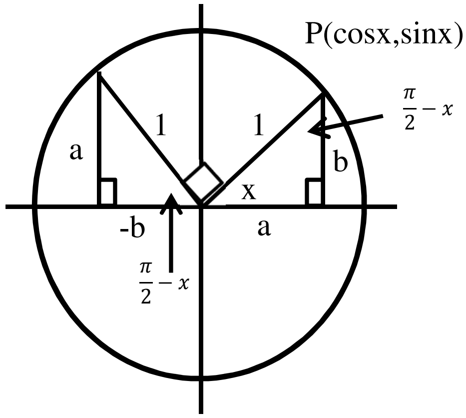
$$\cos\left(\frac{\pi}{2}-x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2}-x\right) = \cot x$$

$$\csc\left(\frac{\pi}{2}-x\right) = \sec x$$

$$\sec\left(\frac{\pi}{2}-x\right) = \csc x$$

$$\cot\left(\frac{\pi}{2}-x\right) = \tan x$$



$\sin x = b$	$\sin\left(\frac{\pi}{2} + x\right) = a$
$\cos x = a$	$\cos\left(\frac{\pi}{2} + x\right) = -b$
$\tan x = \frac{b}{a}$	$\tan\left(\frac{\pi}{2} + x\right) = -\frac{a}{b}$
$\csc x = \frac{1}{b}$	$\csc\left(\frac{\pi}{2} + x\right) = \frac{1}{a}$
$\sec x = \frac{1}{a}$	$\sec\left(\frac{\pi}{2} + x\right) = -\frac{1}{b}$
$\cot x = \frac{a}{b}$	$\cot\left(\frac{\pi}{2} + x\right) = -\frac{b}{a}$

So in Quadrant 2

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\tan\left(\frac{\pi}{2} + x\right) = -\cot x$$

$$\csc\left(\frac{\pi}{2} + x\right) = \sec x$$

$$\sec\left(\frac{\pi}{2} + x\right) = -\csc x$$

$$\cot\left(\frac{\pi}{2} + x\right) = -\tan x$$

Example: Given that $\sin\frac{\pi}{5} \cong 0.5878$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $cos\frac{3\pi}{10}$

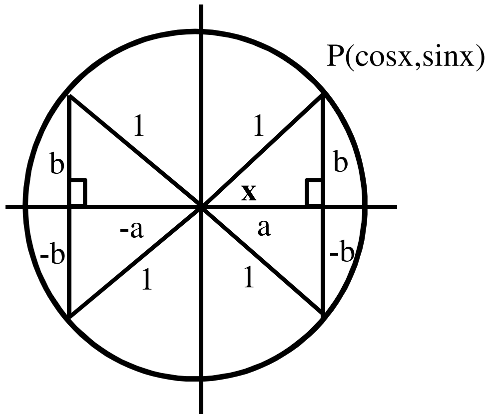
b) $cos\frac{7\pi}{10}$

Example: Given that $\csc\frac{2\pi}{7} \cong 1.2790$, use equivalent trigonometric expressions to determine the $\sec\frac{3\pi}{14}$, to four decimal places.

Example: Given that $\cot\frac{\pi}{6} = \sqrt{3}$, use equivalent trigonometric expressions to show that $\tan\frac{2\pi}{3} = -\sqrt{3}$, to four decimal places.

Example: Given that $\sec b = \csc 1.05$, and that b lies in the first quadrant, use a cofunction identity to determine the measure of angle b, to two decimal places.

More Cofunction Identities



In Quadrant I:

$\sin x = b$
$\cos x = a$
$\tan x = b/a$
$\csc x = 1/b$
$\sec x = 1/a$
$\cot x = a/b$

In Quadrant II:

$sin(\pi - x) = b$
$cos(\pi - x) = -a$
$tan(\pi - x) = -b/a$
$csc(\pi - x) = 1/b$
$sec(\pi - x) = -1/a$
$cot(\pi - x) = -a/b$

In Quadrant III:

$sin(\pi + x) = -b$
$cos(\pi + x) = -a$
$tan(\pi + x) = b/a$
$csc(\pi + x) == 1/b$
$sec(\pi + x) = -1/a$
$cot(\pi + x) = a/b$

In Quadrant IV:

$sin(2\pi - x) = -b$
$cos(2\pi - x) = a$
$tan(2\pi - x) = -b/a$
$csc(2\pi - x) = -1/b$
$sec(2\pi - x) = 1/a$
$cot(2\pi - x) = -a/b$