Section 4.1 Radian Measure

The radian measure of an angle  $\theta$  is defined as the length, a, of the arc that subtends the angle divided by the radius, r, of the circle.



For one complete revolution, the length of the arc equals the circumference of the circle,  $2\pi r$ .

$$\theta = \frac{2\pi r}{r} = 2\pi$$

One complete revolution measures  $2\pi$  radians.

## **Converting Radians to Degrees**

$$2\pi \ radians = 360^{\circ}$$

$$1 radian = \frac{360^{\circ}}{2\pi}$$

$$1 radian = \frac{180^{\circ}}{\pi}$$

Multiply the radians by  $\frac{180^{\circ}}{\pi}$  to determine the equivalent degree measurement.

 $\therefore$  1 radian is approx. 57.3°.

$$1^\circ = \frac{2\pi}{360^\circ}$$

**Converting Degrees to Radians** 

 $360^{\circ} = 2\pi \ radians$ 

$$1^{\circ} = \frac{\pi}{180^{\circ}}$$

Multiply the degrees by  $\frac{\pi}{180^{\circ}}$  to determine the equivalent radian measurement.

 $\therefore$  1° is approx. 0.0175 radians.

## Examples:

- a) Convert 60° to radians. Determine an exact measurement and an approximate measurement.
- b) Using the exact value for  $60^{\circ}$ , determine the radian measure of  $120^{\circ} \& 20^{\circ}$ .

- c) Convert  $\frac{2\pi}{3}$  radians to degrees.
- d) Convert 4.32 radians to degrees.

## Arc Length

To determine the arc length, we use the formula  $\theta = \frac{a}{r}$  and isolate for  $\mathbf{a}, a = r\theta$ . To use the arc length formula  $\theta$  must be measured in radians.

- Example: A circle has a radius of 4.7cm. Determine the length of the arc subtended by each angle.
  - a) 1.7 radians b) 64°

## Angular Velocity of a Rotating Object

The angular velocity of a rotating object is the rate at which the central angle changes with respect to time.

To determine the Angular Velocity of a Rotating Object:

- 1) Multiply the number of revolutions by 3600 or  $2\pi$  depending on whether the answer is required in degrees or radians.
- 2) Divide your answer by the corresponding unit of measurement.

Example: The hard disk in a computer rotates at 7200 revolutions per minute. Determine the angular velocity in:

a) Degrees per second.

b) Radians per second.