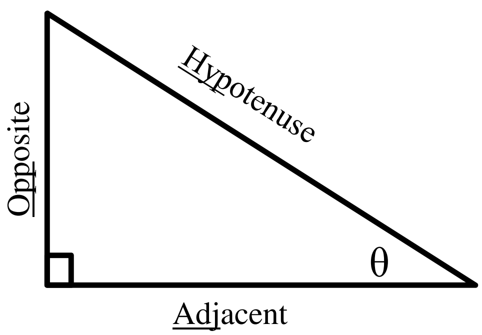
MHF4U Unit 4 Trigonometry

Section	Pages	Questions
Prereq Skills	200-201	#1ac, 2aceg, 3, 5bd, 6aceg, 7, 8, 9, 10, 11ac, 12, 13, 14
4.1	208-210	#1ac, 2ac, 3ac, 4ac, 5ace, 6ace, 7bcf, 8ace, 9, 10, 11, 13ab, 15, 17, 18
4.2	216-219	#1, 3bd, 4ac, 5bc, 6ac, 7, 8, 9, 10, 12, 13, 14, 16a(i), 17a(i), 18a
4.3	225- 227	#1, 2, 3, 4, 5, 6, 9, 10, 11, 14 * use the Unit Circle to derive the six co-function identities for x and (3pi/2-x), and for x and (3pi/2+x)
4.4	232-235	#1ac, 2bd, 3ac, 4a, 5a, 6a, 7a, 8, 9ad, 12, 13, 14, 15 ** Know your proofs of the compound angle formulas!!**
4.5	240-241	#1, 2, 3, 4, 5, 6, 7, 8, 9a, 10, 12, 13, 15, 16
Review (with calc)	244-245 246	# 1ac, 2ac, 3ac, 4ac, 5, 7, 15, 16 # 1, 2, 3, 8
Review (without calc)	244-245 246	# 8, 9, 10, 11, 12a, 13, 14bc, 17, 18, 19a, 20, 23 # 5, 6, 7, 9, 12, 20

Note: Questions with an asterisk* are extra questions that are optional for the daily homework. However, they are potential "extended-type" questions that could be included on a unit test.

Section 4.0 Prerequisite Skills

Primary Trig Ratios

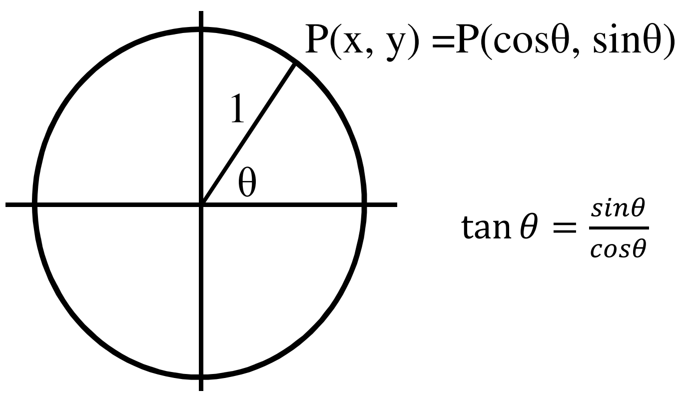


SOH $\sin\theta = \frac{opp}{hyp}$

CAH $\cos\theta = \frac{adj}{hyp}$

TOA $\tan\theta = \frac{opp}{adj}$

Unit Circle



$$\tan\theta = \frac{sin\theta}{cos\theta}$$

CAST Rule

S Sine Ratio is + `ve	A All Trig Ratios are + `ve
T Tan Ratio is + `ve	C Cos Ratio is + `ve

The CAST rule reminds us that for any trig ratio there are two possible angle positions between 0° and 360° where the ratio can give the same result.

To solve for both angles between 0° and 360°:

- $Sin\theta \ = \ k$
$\theta_1 = sin^{-1}(k)$
$\theta_2 = 180° - \ \theta_1$
- $Cos\theta \ = \ k$
$\theta_1 = cos^{-1}(k)$
$\theta_2 = 360° - \ \theta_1$
- $Tan\theta \ = \ k$
$\theta_1 = tan^{-1}(k)$
$\theta_2 = 180° + \ \theta_1$

Examples: An exact value for a trigonometric ratio is given for each angle. Determine the exact values of the other two primary trigonometric ratios.

a) $sin\theta = \frac{5}{13}, 0° \leq \theta \leq 90°$

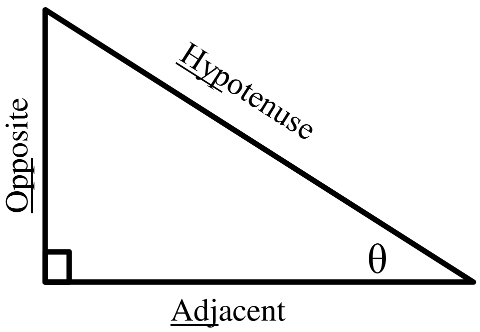
b) $cos\theta = \frac{5}{6}, 270° \leq \theta \leq 360°$

c) $tan\theta = -\frac{15}{17}, 90° \leq \theta \leq 180°$

Examples: Solve the following for both angles between 0° and 360°. Round to the nearest degree.

a) $sin\theta = 0.423$ b) $cos\theta = -0.676$ c) $tan\theta = 4.259$

Reciprocal Trig Ratios (Secondary Trig Ratios)



$$\csc\theta = \frac{hyp}{opp},\ \sec\theta = \frac{hyp}{adj},\ \cot\theta = \frac{adj}{opp}$$

$$\csc\theta = \frac{1}{sin\theta},\ \sec\theta = \frac{1}{cos\theta}, \cot\theta = \frac{1}{tan\theta}$$

** To use our calculators with the reciprocal trig ratios, we must use the reciprocal key, 1/x or x^{-1} along with sine, cosine, or tangent keys.**

Examples: An exact value for a reciprocal trigonometric ratio is given for each angle. Determine the exact values of the other two reciprocal trigonometric ratios.

a) $csc\theta = \frac{12}{7}, 0° \leq \theta \leq 90°$

b) $sec\theta = \frac{17}{15}, 270° \leq \theta \leq 360°$

c) $cot\theta = -\frac{5}{13}, 90° \leq \theta \leq 180°$

Example: Determine the following trig ratios to 3 decimal places.

a) $\csc 36° =$ b) $\sec 80° =$ c) $\cot 52° =$

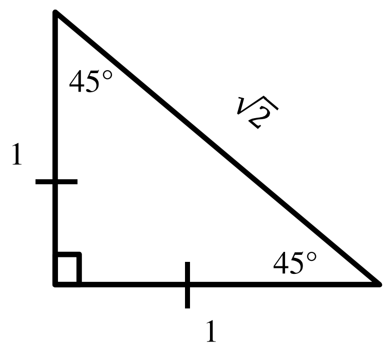
Examples: Solve the following for both angles between 0° and 360°. Round to the nearest degree.

a) $csc\theta = 2.564$ b) $sec\theta = 3.723$ c) $cot\theta = -1.149$

Exact Trigonometric Ratios of Special Angles

A reference angle is the acute angle between the terminal arm and the x-axis.

Angle 45° and its multiples (135°, 225°, 315°)



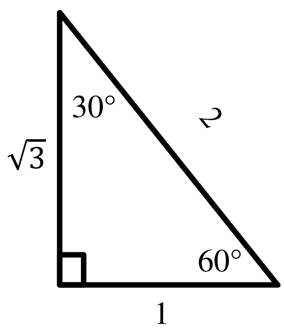
$\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

Use the CAST rule to determine the sign of the trigonometric ratio.

Example: Determine the EXACT VALUES of the sin, cos, and tan of the following angles.

a) $\theta = 135°$ b) $\theta = -45°$ c) $\theta = 225°$

Angle 30º (150º, 210º, 330º) and 60º (120º, 240º, 300º) and their multiples:



$\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

Example: Determine the EXACT VALUES of the sin, cos, and tan of the following angles.

a) θ = 150º b) θ = 240º c) θ = -150º d) θ = -300º

Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Example: Determine the distance between the points (-2, 14) and (5, -9).

Working With Radicals

Radical Sign: the symbol $\sqrt{}$ denotes the positive square root of a number.

Radicand: a number or expression under the radical sign.

Entire Radical: a radical in the form, $\sqrt{n}$, where n > 0. Ex. $\sqrt{24}$

Mixed Radical: a radical in the form $a\sqrt{b}$, where a ≠ 1 or -1, and b > 0. Ex. $2\sqrt{6}$

Multiplication Property of Radicals: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, where a ≥ 0, b ≥ 0

* **Change Entire Radical to Mixed Radicals/ Write Radicals in Simplest Form**:

1) Look for the largest perfect square factor
2) Use the multiplication property to write the radical in simplest form (i.e. Write as a "mixed radical").

Example: Express the following as a mixed radical in simplest form.

a) $\sqrt{32}$ b) $\sqrt{40}$ c) $\sqrt{50}$ d) $\sqrt{300}$

Adding and Subtracting Radicals

To add and subtract radicals, you must have **like radicals**.

Like radicals have the **same radicand**, i.e. same number under the radical sign

For example, $\sqrt{6}$ and $3\sqrt{6}$ are like radicals, whereas $\sqrt{10}$ and $\sqrt{7}$ are NOT like radicals

Sometimes we can make like radicals by rewriting radicals in simplest form.

Ex. $5\sqrt{8}$ and $2\sqrt{18}$

$$\left.\begin{array}{l} \rightarrow\ 5\sqrt{8} = 5\sqrt{4x2} = 5\sqrt{4}\sqrt{2} = 10\sqrt{2} \\ \rightarrow\ 2\sqrt{18} = 2\sqrt{9x2} = 2\sqrt{9}\sqrt{2} = 6\sqrt{2} \end{array}\right\} \text{Now like radicals}$$

When adding and subtracting like radicals:

1) Add/subtract the coefficients in front of the radical. *The number under the radical sign stays the same. (The radicand remains constant)

2) Ensure all radicals in your final answer are written in simplest form.

Examples: Simplify and collect like radicals.

a) $\sqrt{18}+\sqrt{75}-\sqrt{27}+\sqrt{8}$

b) $6\sqrt{20}+4\sqrt{54}-5\sqrt{24}-2\sqrt{125}$

c) $\frac{1}{4}\sqrt{12}-3\sqrt{28}+\frac{5}{8}\sqrt{48}+\frac{2}{3}\sqrt{63}$

Multiplication and Division Properties of Radicals:

$\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

where a ≥ 0, b ≥ 0

$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

where a ≥ 0, b ≥ 0

Example: Simplify the following radicals

a) $\sqrt{11} \times \sqrt{6}$

b) $\frac{\sqrt{35}}{\sqrt{5}}$

c) $2\sqrt{13} \times 3\sqrt{2}$

d) $\frac{4\sqrt{10}}{2\sqrt{2}}$

e) $\frac{8-\sqrt{20}}{2}$

f) $3\sqrt{2}\left(2\sqrt{3}-5\right)-\sqrt{6}\left(1-4\sqrt{3}\right)$

g) $\left(2+4\sqrt{3}\right)\left(2-4\sqrt{3}\right)$

h) $\left(\sqrt{3}+6\right)\left(5-\sqrt{3}\right)$

Example: Determine the exact perimeter and exact area of a rectangle with length 7 cm and diagonal 9 cm.

Rationalizing the Denominator

Fractions should not be left with radicals in the denominator (just like how we should not have decimals in fractions). Rationalizing the denominator is the process in which we remove the radical from the denominator (similar to finding the common denominator when working with fractions).

If there is only the radical in the denominator:

1. Multiply the numerator and denominator by the radical. $\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$

2. Simplify the numerator and the denominator. $\frac{a\sqrt{b}}{\sqrt{b^2}} = \frac{a\sqrt{b}}{b}$

Example: Rationalize the following denominators

a) $\frac{2}{\sqrt{3}}$ b) $\frac{3\sqrt{2}}{\sqrt{3}}$ c) $\frac{2+\sqrt{3}}{\sqrt{2}}$