Section 3.3 Rational Functions of the Form $f(x) = \frac{ax+b}{cx+d}$

In this section you will look at polynomial functions in which both the numerator and denominator are linear expressions.

Because there is a variable in both the numerator and denominator, there are effects on both the vertical and horizontal asymptotes and as a result the domain and range.

A rational function of the form $f(x) = \frac{ax+b}{cx+d}$ has the following key features:

- The equation of the vertical asymptote can be found by setting the denominator equal to zero and solving for x, provided the numerator does not have the same zero.

- The equation of the horizontal asymptote can be found by dividing each term in both the numerator and the denominator by x and investigating the behaviour of the function as $x \to \pm\infty$.

- The **b** constant acts to stretch the curve, but has no effect on the asymptotes, domain, or range.

- The **d** constant shifts the vertical asymptote.

- The two branches of the graph of the function are equidistant from the point of intersection of the vertical and horizontal asymptotes.

Example: Consider the function $f(x) = \frac{x+4}{x-2}$

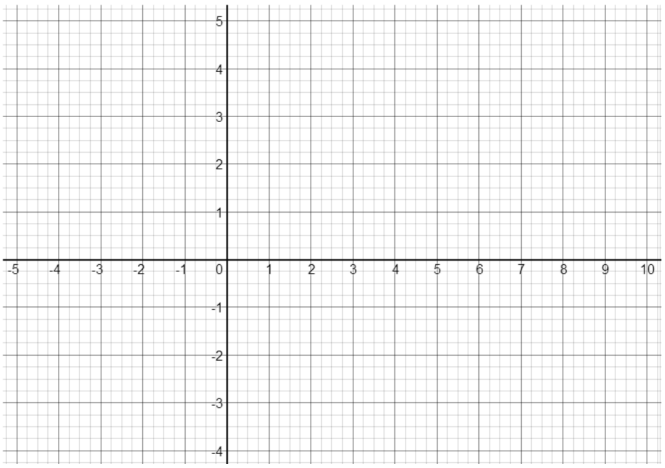
a) Determine the equation of the vertical asymptote.

b) Determine the equation of the horizontal asymptote.
(*divide each term by x and simplify*)

c) Determine the x-and y-intercepts.

d) State the domain and range.

e) Sketch a graph of the function and label all important points.



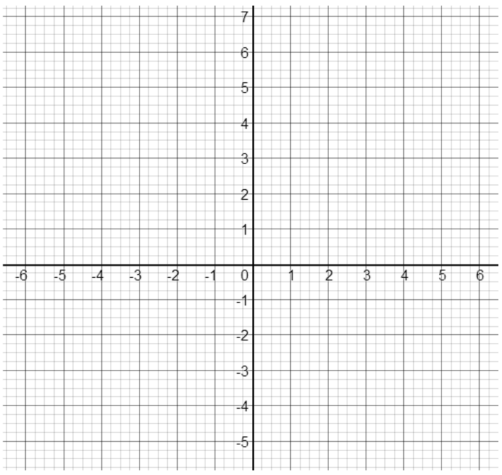
f) Complete a table to summarize the intervals of increase and decrease.
(*Include the vertical asymptotes and the x-intercepts.*)

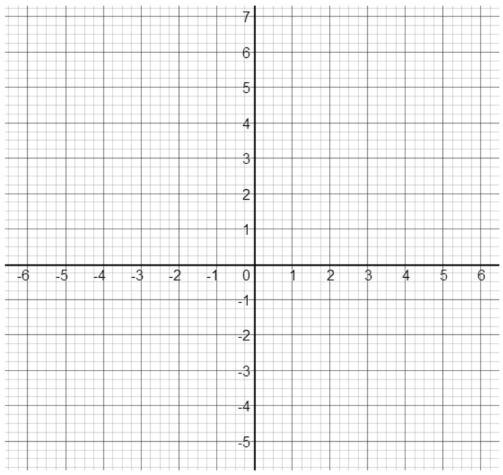
Interval				
Sign of $f(x)$				
Sign of Slope				

Examples: Compare the effects of the functions by graphing all 3 functions. Find the asymptotes and intercepts to help graph each function. State the domain and range of each function.

a) $f(x) = \frac{x-1}{2x+3}$

b) $g(x) = \frac{x-2}{2x+3}$





Example: Write an equation for a rational function whose graph has all of the indicated features.

→ x-intercept of $\frac{4}{7}$ → y-intercept of -2

→ horizontal asymptote at $y = \frac{7}{3}$ → vertical asymptote at $x = -\frac{2}{3}$

Example: Write an equation for the rational function shown on the graph below.