Section 3.2 Reciprocal of a Quadratic Function

Rational functions can be analyzed using the following key features:

- Asymptotes
- Intercepts
- Slope (positive or negative, increasing or decreasing)
- Domain and Range
- Positive and Negative Intervals

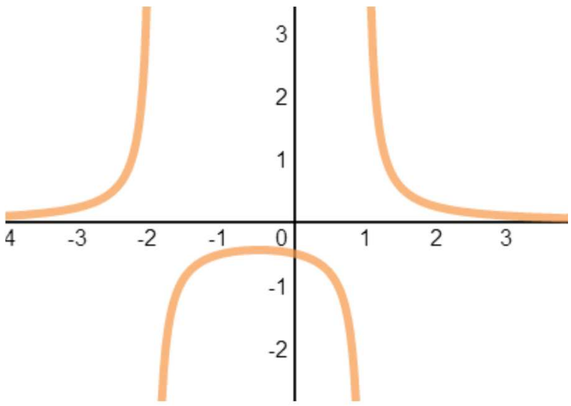
To find the domain of a quadratic function,

1. Factor the denominator, set each factor = 0 and solve each one
   (ie. Determine the roots of the quadratic denominator)

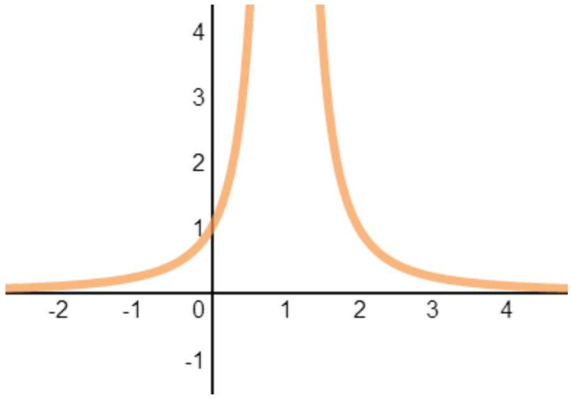
2. The solutions are the restrictions of the function and hence the values of x for which the function is not defined.

The equations of the vertical asymptotes are the restrictions for x with an = instead of a ≠.

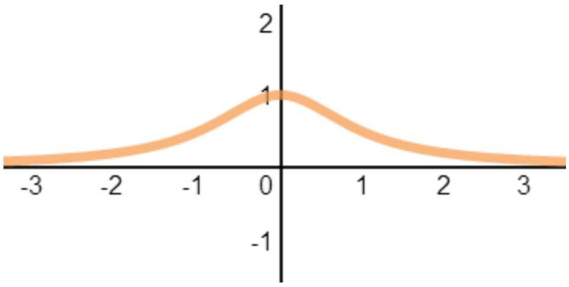
Reciprocals of quadratic functions with **two zeros** have three parts, with the middle one being a parabola. Therefore, the parabola part has a maximum or minimum point. This point is equidistant from the two vertical asymptotes and it is the x-coordinate of the vertex of the parabolic branch.



Reciprocals of quadratic functions with **only one zero** have two parts with only one vertical asymptote.



Reciprocals of quadratic functions with no real roots have only one part with no vertical asymptote.



All reciprocals of quadratic functions have a horizontal asymptote in the x-axis, at y = 0.

The behaviour near asymptotes is similar to that of reciprocals of linear functions.

All of the behaviours listed above can be predicted by analyzing the roots of the quadratic relation in the denominator.

Example: Consider the function $f(x) = \frac{2}{x^2-4}$

a) Determine the domain.

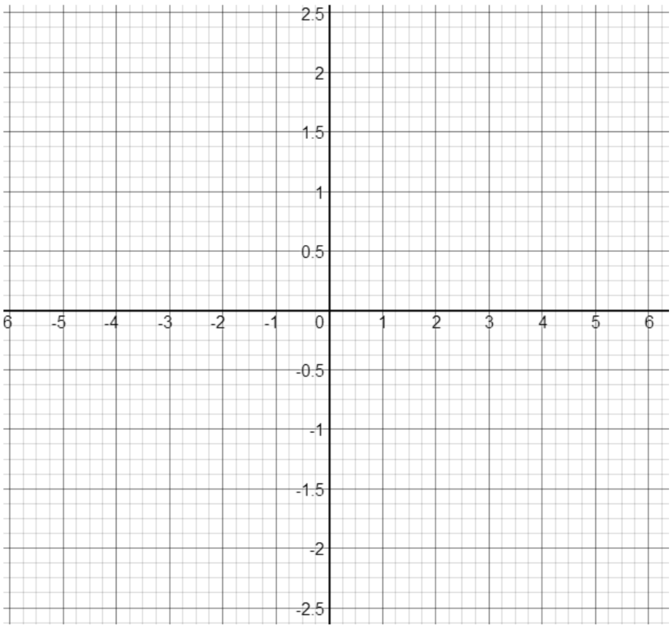
b) State the equation of the asymptotes.

c) Describe the behaviour of the function near the asymptotes and the end behaviour.

d) Determine the x and y intercepts

e) Determine the max/min point of the parabolic branch of the function.

f) Sketch the graph of the function.



g) State the range.

Example: Consider the function $f(x) = \frac{-10}{x^2-4x-5}$

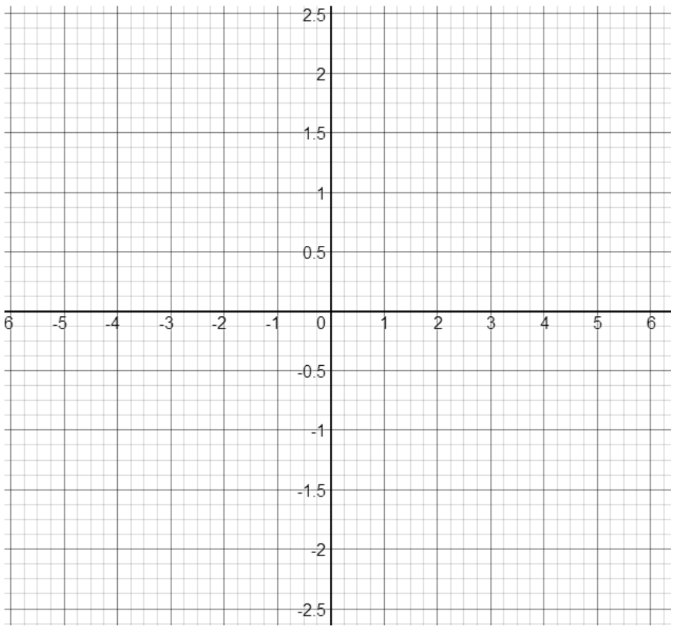
a) Determine the domain.

b) State the equations of the asymptotes.

c) Determine the intercepts.

d) Determine the min/max point of the parabolic branch of the function.

e) Sketch a graph of the function and label all important points.



f) State the range.

g) Complete the table below to summarize the intervals of increase and decrease.
Use the values of x at the asymptotes and vertex for your intervals.

Interval					
Sign of f(x)					
Sign of Slope Function is increasing (m) or decreasing (-m)					
Change in Slope (slope is increasing +) (slope is decreasing -)					

Example: For the following functions,

a) Determine the domain.
b) State the equation(s) of the asymptote(s).
c) Determine the intercepts.
d) Sketch a graph.
e) State the range.

i) $f(x) = \frac{5}{x^2+9}$

ii) $g(x) = -\frac{4}{x^2+2x+1}$

Example: Each function described below is the reciprocal of a quadratic function. Write an equation to represent each function.

a) The horizontal asymptote is $y = 0$. The vertical asymptotes are $x = -1$ and $x = 3$. And $f(x) < 0$ for the intervals $x < -1$ and $x > 3$.

b) The horizontal asymptote is $y = 0$. There is no vertical asymptote. Domain is $x \in R$. The maximum point is $(0, 1/5)$.