The reciprocal of a linear function has the form $f(x) = \frac{a}{kx-d} + c = \frac{1}{k\left(x-\frac{d}{k}\right)} + c$

The **restriction on the domain** of a reciprocal linear function can be determined by finding the value of x that makes the **denominator equal to zero**, that is $x = \frac{d}{k}$. Therefore, the domain of a reciprocal linear function is $\{x \in R, x \neq d/k\}$

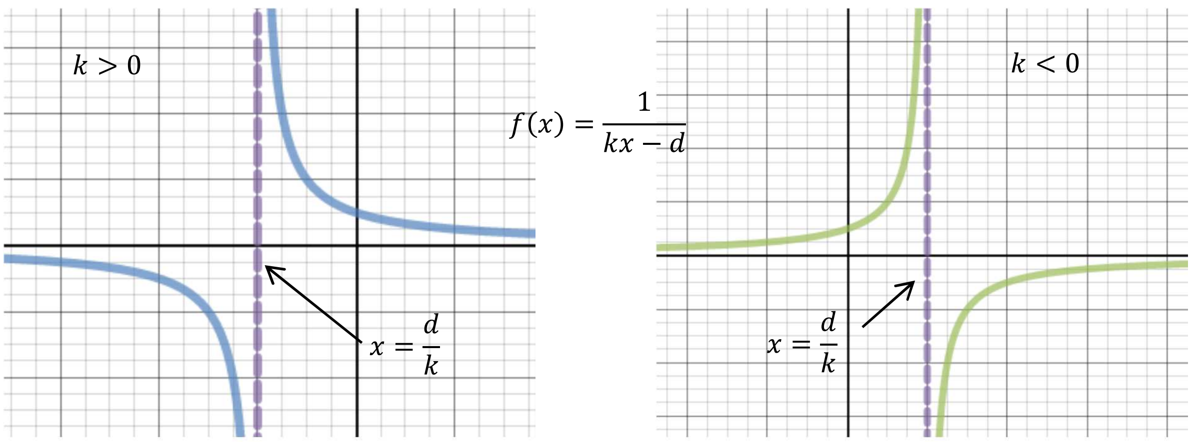
Asymptotes

The **vertical asymptote** of a reciprocal linear function occurs when x =d/k.

$x \to x^+$ means "as x approaches a from the right"

$x \to x^-$ means "as x approaches a from the left"

The **horizontal asymptote** of a reciprocal linear function of the form $f(x) = \frac{1}{kx-d} + c$ has equation y = c.



If $k > 0$, the left branch of a reciprocal linear function has a negative, decreasing slope, and the right branch has a negative, increasing slope.

If $k < 0$, the left branch of a reciprocal linear function has a positive, increasing slope, and the right branch has a positive, decreasing slope.

Example: Consider the function $f(x) = \frac{1}{x+2}$

a) State the domain

b) Make a sketch of the function

c) Describe the behaviour of the function near the vertical asymptote.

$\therefore As\ x \rightarrow\ 2^{-}\ f(x) \rightarrow$, $As\ x \rightarrow\ 2^{+}\ f(x) \rightarrow$

d) Describe the end behaviour (as x approaches negative and positive infinity)

$\therefore As\ x \rightarrow\ -\infty\ f(x) \rightarrow$, $As\ x \rightarrow\ +\infty\ f(x) \rightarrow$

e) State the Range

f) Describe the intervals where the slope is increasing and the intervals where the slope is decreasing in the two branches of the rational function.

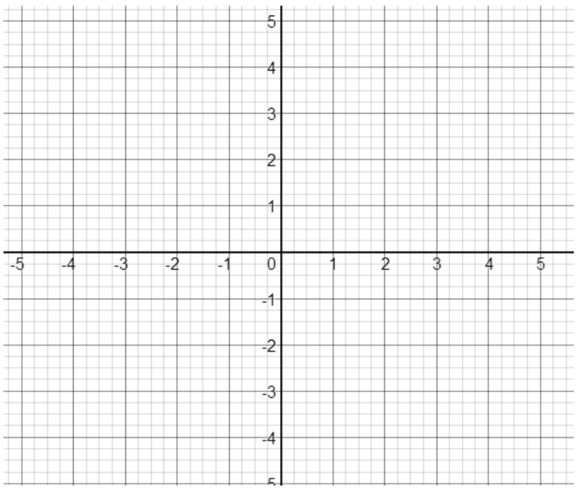
Example: Determine the x-intercepts and y-intercepts of the function $g(x) = \frac{3}{x+4}$

Example: Determine the equation in the form $f(x) = \frac{1}{kx-d}$ for the function with a vertical asymptote at x = -2 and a y-intercept at -1/10.

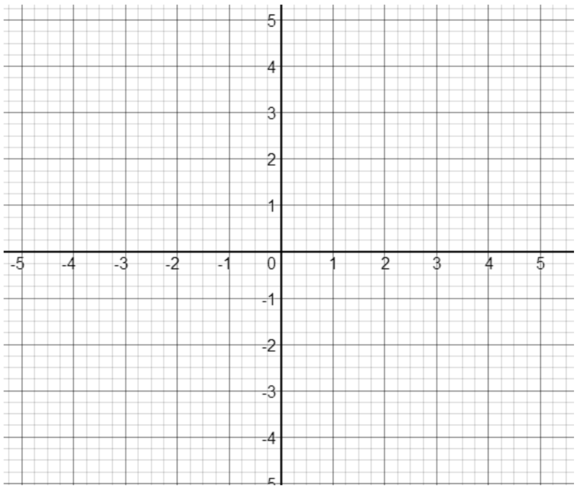
Example: For each reciprocal function

i) write an equation to represent the vertical asymptote
ii) write an equation to represent the horizontal asymptote
iii) determine the x-and y-intercepts
iv) state the domain and range
v) sketch a graph
vi) describe the intervals where the slope is increasing and where it is decreasing

a) $f(x) = \frac{3}{x-2}$



b) $g(x) = -\frac{1}{2x+5}$



c) $h(x) = \frac{2}{1-x}$