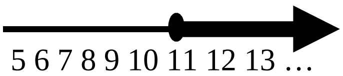
Section 2.6 Solve Factorable Polynomial Inequalities Algebraically

Solving Linear Inequalities

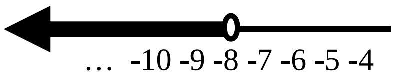
- Solve as you would a regular equation.
- Remember to flip the inequality sign when dividing/multiplying by a negative number.

Example: Solve each linear inequality and graph your solution on a number.

a) $x - 7 \geq 4$
$x \geq 4 + 7$
$x \geq 11$



b) $-4 - 2x > 12$
$- 2x > 12 + 4$
$- 2x > 16$
$x < -8$



Solve Polynomial Inequalities

- Factorable inequalities can be solved algebraically by factoring the polynomial, if necessary, and determining the zeros/roots of the function.

Then...

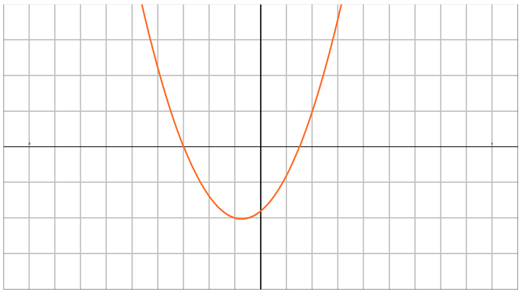
1. Consider all cases, OR
2. Use intervals and then test values in each interval

- Tables and number lines can help organize intervals to provide a visual clue to solutions.

Example: Solve the inequality using cases and intervals.

a) $(x + 3)(2x - 3) > 0$

Roots x = - 3, 3/2



The polynomial is already in factored form so we have saved a little work. This will not always be true.

We will start with the pure **Algebra** based solution.

- Step 1 – determine the number of possible cases for the inequalities

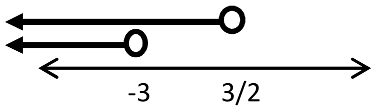
  We have two brackets being multiplied with the goal being to determine when this function will be greater than zero. i.e when is the function positive

  The product will be positive when both brackets have a positive value (case 1) or when both brackets have negative values (case 2).

  So we have two cases

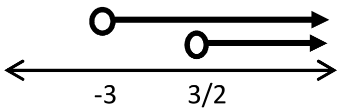
- Step 2 – Solve for both cases (determine when are both true)

Case 1	$x + 3 > 0$	$2x - 3 > 0$
	$x > -3$	$2x > 3$
		$x > 3/2$



Both will be positive for numbers less than – 3

Case 2	$x + 3 < 0$	$2x - 3 < 0$
	$x < -3$	$2x < 3$
		$x < 3/2$



Both will be negative for numbers greater than 3/2

- Step 3 Write your concluding inequality statement

  $\therefore (x + 3)(2x - 3) > 0$ when $x < -3$ & $x > 3/2$

Let’s try this problem a second way using the interval method

Intervals: Start with the idea that this function has the **potential** to change from positive to negative values at the roots. We say potential because it could just touch the axis and bend back.

- We will create a table to discuss all regions for the function in space,
- We will test values in these regions in each of the factors to determine the sign of the function.

- We know the roots of this function are at x = -3, 3/2 so let us discuss the interval before -3, between -3 and 3/2, and after 3/2.

- We will still use the logic from the algebra solution, that both factors must either be positive or negative to provide a result that is > 0.

Interval	x < -3	-3 < x < 3/2	x > 3/2
Factors	Try – 4	Try 1	Try 4
(x + 3)	(-4 + 3) = -1 Sign (-)	(1 + 3) = 4 Sign (+)	(4 + 3) = 7 Sign (+)
(2x – 3)	[2(-4) – 3] = -11 Sign (-)	[2(1) – 3] = -1 Sign (-)	[2(4) – 3] = 5 Sign (+)
Result (x+3)(2x -3)	(+)	(-)	(+)

$\therefore (x + 3)(2x - 3) > 0$ when x < -3 & x > 3/2

Example: Solve the inequality using cases and intervals.

b) $-2x^3 - 6x^2 + 12x + 16 \leq 0$

Example: The price, P, in dollars, of a stock t years after 2000 can be modelled by the function $P(t) = 0.4t^3 - 4.4t^2 + 11.2t$. When will the price of the stock be more than \$36?