MHF4U Unit 2 Polynomial Equation and Inequalities

Section	Pages	Questions
Prereq Skills	82 - 83	# 1ac, 2ace, 3adf, 4, 5, 6ace, 7ac, 8ace, 9ac
2.1	91 – 93	#1, 2, 3bdf, 4ac, 5, 6, 7ab, 8c, 9ad, 10, 12, 15a, 16, 20*, 22*
2.2	102 – 103	#1bc, 2ac, 3ac, 4ace, 5ac, 6aceg, 7aceg, 9, 10, 11acf, 12a(i, iii)d(ii,iv), 13a(ii,iii)d(i,iii), 15, 17*, 18*, 20*
2.3	110 – 111	#1ace, 2ace, 3aceg, 4aceg. 5. 6aceg. 7aceg. 8ace. 9ace (inclass), 10, 14, 17*, 18*, 20*
2.4	119 – 122	#1, 2, 3, 4(don't graph), 5d, 8, 10-16
2.5	129 – 130	#1-5
2.6	138 –139	#1ace, 2, 3ac, 4ab, 5ad, 6ac, 7ac, 8, 9
Review	140 – 141 142 - 143	#1, 2, 3, 4ac, 5, 6, 7, 8, 9, 10, 12, 13, 14, 17ac, 18ab# 8abd, 13 #1-5, 11, 12

Note: Questions with an asterisk* are extra questions that are optional for the daily homework. However, they are potential "extended-type" questions that could be included on a unit test.

Section 2.1 The Remainder Theorem

In math it can be very helpful to know if one object will fit into another. Whether it can fit perfectly, or if there will be something left over.

In elementary school you were taught that to solve this problem we divided the size of the larger object by smaller object using long division. If the remainder was zero, it fit perfectly. If it was not zero, there was some amount left over.

Long Division a) $3\overline{)456}$ b)) $6\overline{)456}$

Polynomials can be divided the same way.

Ex. $x-2\overline{)2x^3-3x^2-3x+2}$

Focus only on the leading term in the divisor
What do we have to multiply x by to get $2x^3$?
Answer: $2x^2$. We put this on the top line, and multiply it with the divisor

$$
\begin{array}{r}
2x^2 \\
x-2\overline{)2x^3-3x^2-3x+2} \\
\underline{2x^3-4x^2} \\
x^2-3x
\end{array}
$$

Subtract the $2x^3-4x^2$ from the numbers above & bring the next term down (the -3x)

$$
\begin{array}{r}
2x^2+x-1 \\
x-2\overline{)2x^3-3x^2-3x+2} \\
\underline{2x^3-4x^2} \\
x^2-3x \\
\underline{x^2-2x} \\
-x+2 \\
\underline{-x+2} \\
0
\end{array}
$$

What do you have to multiply x by to get x^2?
Answer: x. We put this on the top line, and multiply it with the divisor

Subtract the x^2-2x from the numbers above & bring the next term down (the 2)

What do you have to multiply x by to get $-x$?
Answer: -1. We put this on the top line, and multiply it with the divisor

Subtract the $-x+2$ from the numbers above & your done

In this case the remainder was zero, it fit perfectly. <u>This will not always be the case.</u>

The result of the division of a polynomial function P(x) by a binomial of the form x - b is

$$\underbrace{\frac{P(x)}{x-b} = Q(x) + \frac{R}{x-b}}_{\text{Quotient Form}}$$ where Q(x) is the quotient and R is the remainder.

The corresponding statement, that can be used to check the division, is

$$P(x) = (x - b)Q(x) + R$$

So to check the result of a division, use: divisor x quotient + remainder = dividend

Examples: Divide the following polynomials. Express the result in a quotient form. Identify any restrictions on the variable. Write the corresponding statement that can be used to check the division. Verify your answer.

Dividing a Polynomial by a Binomial of the Form $x - b$

a) $(-3x^2 + 2x^3 + 8x - 12) \div (x - 1)$

Note: If the quotient (the polynomial that you are dividing) has a term in x missing, add a term by placing a zero in front of it. For example, if you are dividing $x^3 + x - 4$ by something, rewrite it as $x^3 + 0x^2 + x - 4$.

Dividing a Polynomial by a Binomial of the Form $ax - b$

b) $(4x^3 + 9x - 12) \div (2x + 1)$

Apply Long Division to Solve for Dimensions

Example: The volume, V, in cubic centimeters, of a rectangular box is given by $V(x) = x^3 + 7x^2 + 14x + 8$. Determine expressions for possible dimensions of the box if the height, h, in centimeters, is given by $x + 2$.

The Remainder Theorem

When dividing one algebraic expression by another, more often than not there will be a remainder. It is often useful to know what this remainder is and, yes, it can be calculated without going through the process of dividing as before.

The rule is:

When a polynomial function $P(x)$ is divided by $x - b$, the remainder is $P(b)$; and when it is divided by $ax - b$, the remainder is $P(b/a)$, where a and b are integers, and a $\neq$ 0.

Let's check our last two examples to see how this would work.

$P(x)$ is divided by $x - b$

a) $(-3x^2 + 2x^3 + 8x - 12) \div (x - 1)$

To determine the remainder in advance we could have just subbed $x = 1$ into the function.

$$P(b) = -3(1)^2 + 2(1)^3 + 8(1) - 12$$
$$P(b) = -3 + 2 + 8 - 12$$
$$P(b) = -5$$

$P(x)$ is divided by $ax - b$

b) $(4x^3 + 9x - 12) \div (2x + 1)$

To determine the remainder in advance we could have just subbed $x = -\frac{1}{2}$ into the function.

$$P(b/a) = 4\left(-\frac{1}{2}\right)^3 + 9\left(-\frac{1}{2}\right) - 12$$

$$P(b/a) = 4\left(-\frac{1}{8}\right) + \left(-\frac{9}{2}\right) - 12$$

$$P(b/a) = -\frac{1}{2} - \frac{9}{2} - 12$$

$$P(b/a) = -\frac{10}{2} - 12$$

$$P(b/a) = -5 - 12$$

$$P(b/a) = -17$$

Examples:

a) Use the remainder theorem to determine the remainder when $P(x) = 2x^3 + x^2 - 3x - 6$ is divided by $x + 1$. Verify your answer using long division.

b) Use the remainder theorem to determine the remainder when $P(x) = 2x^3 + x^2 - 3x - 6$ is divided by $2x - 3$. Verify your answer using long division.

c) Determine the value of k such that when $P(x) = 3x^4 + kx^3 - 7x - 10$ is divided by $x - 2$, the remainder is 8.