

Section 1.6

Slopes of Tangents and Instantaneous Rate of Change

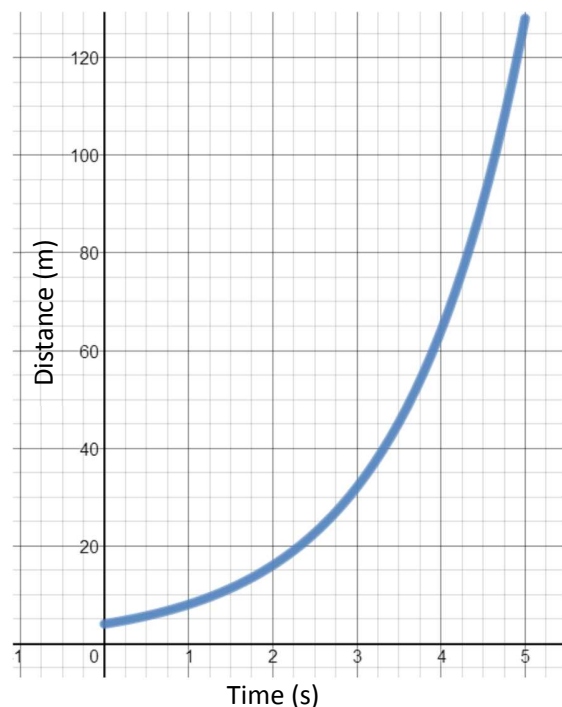
- A tangent to a curve is a line that intersects a curve at exactly one point.
 - An instantaneous rate of change corresponds to the slope of a tangent at a point on a curve.
 - An approximate value for an instantaneous rate of change at a point may be determined using....
1. a **graph**, either by estimating the slope of a secant passing through that point OR by sketching the tangent and estimating the slope between the tangent point and a second point on the approximate tangent line.
 2. a **table of values**, by estimating the slope between the point and a nearby point in the table.
 3. an **equation**, by estimating the slope using a very short interval between the tangent point and a second point found using the equation.

Relationship Between the Slope of Secants and the Slope of a Tangent

- As a point Q becomes very close to a tangent point P, the slope of the secant line becomes closer to (approaches) the slope of the tangent line.
- Often an arrow is used to denote the word "approaches". So, the above statement may be written as follows:
As $Q \rightarrow P$, the slope of the secant $PQ \rightarrow$ the slope of the tangent at P.
- Thus, the average rate of change between P and Q becomes closer to the value of the instantaneous rate of change at P.

Example: The graph shows the approximate distance travelled by a parachutist in the first 5 seconds after jumping out of a helicopter. How fast was the parachutist travelling 2s after jumping out of the helicopter?

- i) Use the slope of a secant.
- ii) Use 2 points on an approximate tangent.



Example: In the table below, the distance of the parachutist in the previous example is recorded at 0.5 second intervals. Estimate the parachutist's velocity at 2 sec.

Time (s)	Distance (m)
0	0
0.5	1.25
1	5
1.5	11.25
2	20
2.5	31.25
3	45
3.5	61.25
4	80

Example: The functions $d(t) = 5t^2$ can be used to approximate the distance travelled by the parachutist in the previous two examples. Use the equation to estimate the velocity of the parachutist after 2 seconds.

Interval	Δd	Δt	$\Delta d / \Delta t$