Section 1.5 Slopes of Secants and Average Rate of Change

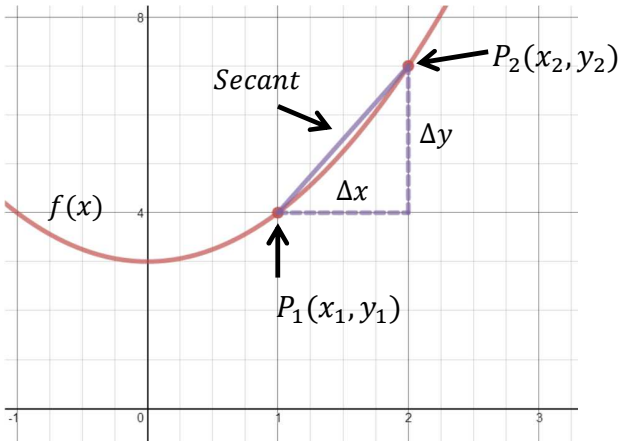
Key Concepts

- A secant is a straight line that connects two points on a curve.
- Rate of Change (Slope) is a measure of how quickly one quantity (the dependent variable) changes with respect to another quantity (the independent variable).

There are two types of rates of change, average and instantaneous.

Average rates of change

- represent the rate of change over a specified interval corresponding to the slope of a secant between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a curve

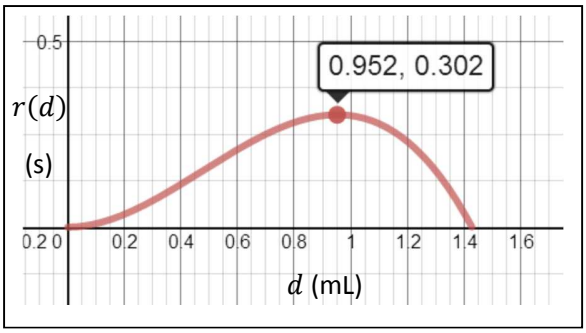


$$\text{Avg. Rate of Change} = \frac{\Delta y}{\Delta x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

- An average rate of change can be determined by calculating the slope between two points given in a table of values or by using an equation.

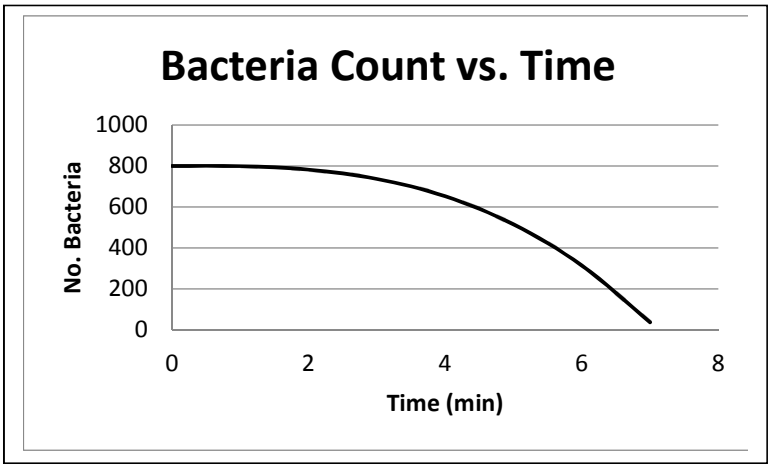
Example: A medical researcher establishes that a patient's reaction time, r, in minutes, to a dose of a particular drug is $r(d) = -0.7d^3 + d^2$, where d is the amount of the drug, in mL, that is absorbed into the patient's blood, and $d \in [0, 1.428]$.

Use the graph to describe when the rate of change is positive, when it is zero, and when it is negative.



Example: A new antibacterial spray is tested on a bacterial culture. The table shows the population, P, of the bacterial culture t minutes after the spray is applied.

t (min)	P
0	800
1	799
2	782
3	737
4	652
5	515
6	314
7	37



How can you tell the average rate of change is negative by examining

i) the table of values

ii) the graph

Determine the average rate of change of the number of bacteria over the entire time period shown in the table. Interpret this value for this situation.

Compare the average rates of change of the number of bacteria in the first 3 minutes and the last 3 minutes. Explain any similarities and differences. Draw the secants on the graph and label them AB and CD respectively.

Example: A football is kicked into the air such that its height, h, in metres after t seconds is modeled by the function $h(t) = -4.9t^2 + 14t + 1$. Determine the average rate of change of the height of the ball for each time interval. Interpret these values.

a) [0, 0.5]

b) [2, 2.5]