Section 1.4 Transformations

Transformations

The graph of a function of the form $f(x) = a[k(x - d)]^n + c$ is obtained by applying the transformations to the graph of the power function $f(x) = x^n$, where n is a non-negative integer, $n \in N$.

The parameters **a, k, d**, and **c** correspond to the following transformations:

- **a** corresponds to a vertical stretch (if **a** > 1) or compression (if 0 < **a** < 1) by a factor of **a** if **a** < 0, a reflection in the x-axis.

- **k** corresponds to a horizontal stretch (if 0 < **k** < 1) or compression (if **k** > 1) by a factor of 1/**k** if **k** < 0, a reflection in the y-axis.

- **c** corresponds to a vertical translation up (if **c** > 0) or down (if **c** < 0)

- **d** corresponds to a horizontal translation to the left (if **d** < 0) or right (if **d** > 0)

An accurate sketch of the transformed graph is obtained by applying transformations represented by **a** and **k** before the transformations represented by **c** and **d**.

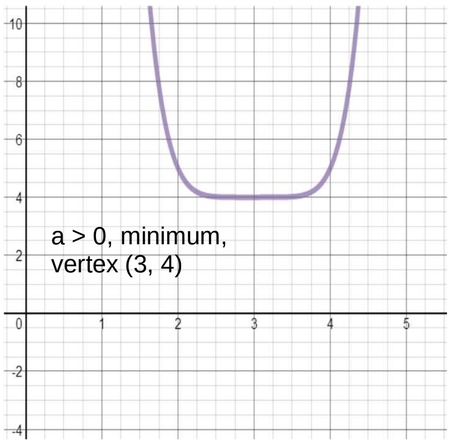
R.S.T = **R**eflect **S**tretch(Compress) **T**ranslate

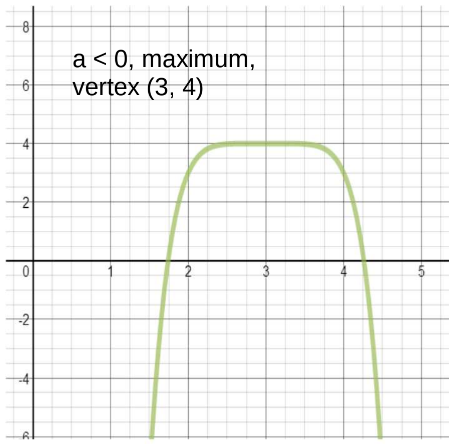
Even-Degree Polynomial Functions

When **n is even**, the graph of the polynomial function $f(x) = a[k(x - d)]^n + c$ is an even-degree function and has a vertex at (**d, c**). The axis of symmetry is x = d.

- For **a** > 0, the graph opens upward. Therefore the graph extends from Q2 to Q1.
- The vertex is the minimum point on the graph and **c** is the minimum value.
  The range of the function is $\{y \in R \mid y \geq \boldsymbol{c}\}$.

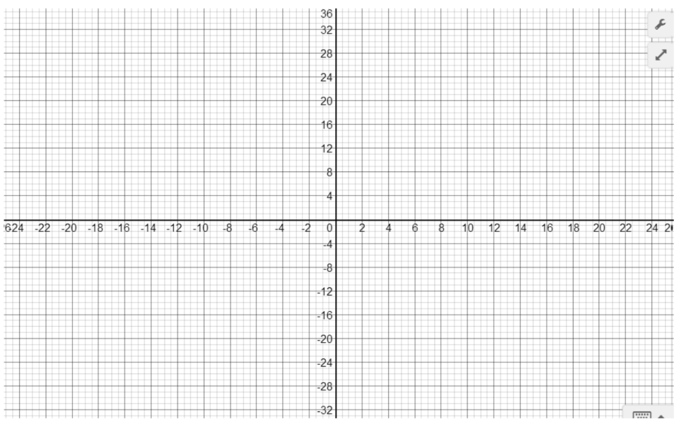
- For **a** < 0, the graph opens downward. Therefore the graph extends from Q3 to Q4.
- The vertex is the maximum point on the graph and **c** is the maximum value.
  The range of the function is $\{ y \in R \mid y \leq \boldsymbol{c}\}$.





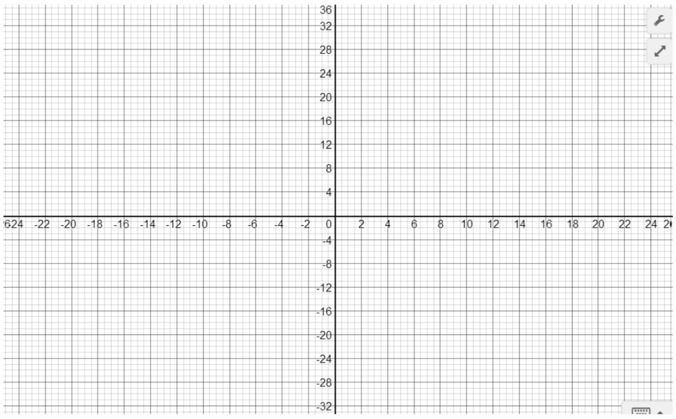
Example: Given a base function of $f(x) = x^4$, state the parameters and the corresponding transformations required to obtain the graph of $g(x) = -2\left[\frac{1}{3}(x-4)\right]^4 - 1$

Sketch the graph and state the domain, range, vertex, and equation of the axis of symmetry for the transformed function.



Example: Given a base function of $f(x) = x^3$, state the parameters and the corresponding transformations required to obtain the graph of $g(x) = 3[-2(x+1)]^3 + 5$

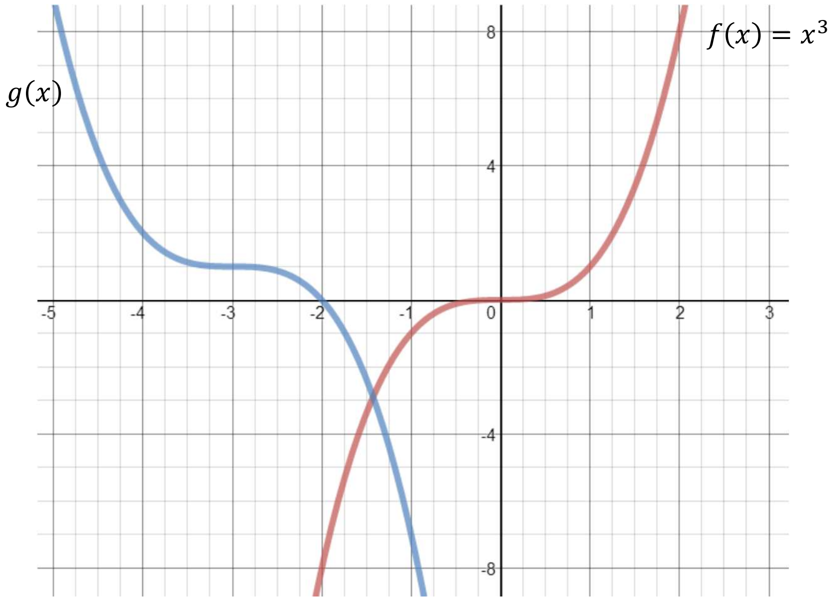
Sketch the graph and state the domain, range, vertex, and equation of the axis of symmetry for the transformed function.



Example: Describe the transformations that must be applied to the graph of each power function, $f(x)$, to obtain the transformed function. Then, write the corresponding equation of the transformed function. State the domain and range for each transformed function, and state the vertex and equation of the axis of symmetry for any even functions.

a) $f(x) = x^5, g(x) = -\frac{1}{2}f(3x-6)$ b) $f(x) = x^6, g(x) = \frac{4}{3}f\left[-\frac{1}{3}(x+5)\right] - 1$

Example: Transformations are applied to the power function $f(x) = x^3$ to obtain the resulting graph. Determine an equation for the transformed function, g(x). State its domain and range.



Example: Write an equation for the function that would result from the given transformations. Then state the domain and range, the vertex, and the equation of the axis of symmetry. The function $f(x) = x^4$ is reflected in the x-axis, stretched vertically by a factor of 3.5, compressed horizontally by a factor of 2/3, translated 6 units to the right, and 7 units down.