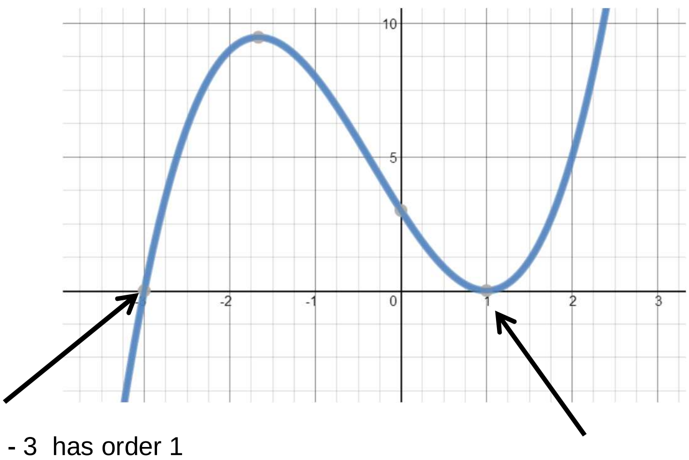
The graph of a polynomial function can be sketched using the **x-intercepts**, the **degree** of the function, and the **sign of the leading coefficient**.

- The x-intercepts of the graph of a polynomial function are the roots of the corresponding polynomial equation.
- When a polynomial function is in factored form, the zeros can be easily determined from the factors. When a factor is repeated n times, the corresponding zero has order n.

Example: The function $f(x) = (x - 1)^2(x + 3)$ has zeros at 1 and -3. However since the factor $(x - 1)$ is repeated twice, $x = 1$ is a zero of order 2.

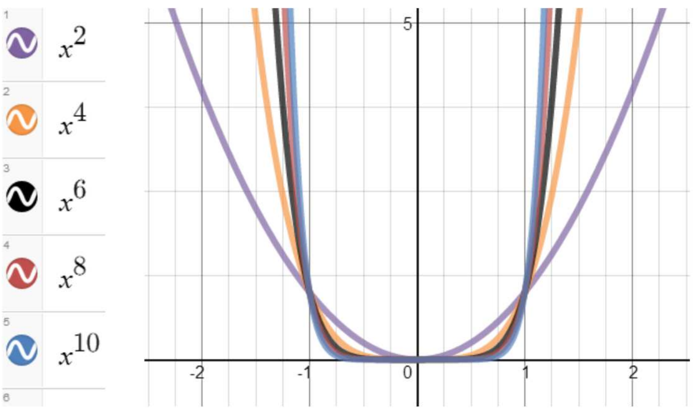
- The graph of a polynomial function changes sign (y coordinates change sign) only at zeros of odd order. At zeros of even order, the graph touches but does not cross the x-axis.



x= - 3 has order 1
The sign of the function changes
(the function will cross the x-axis)

x= 1 has order 2
The sign of the function does not change
(the function will not cross the x-axis)

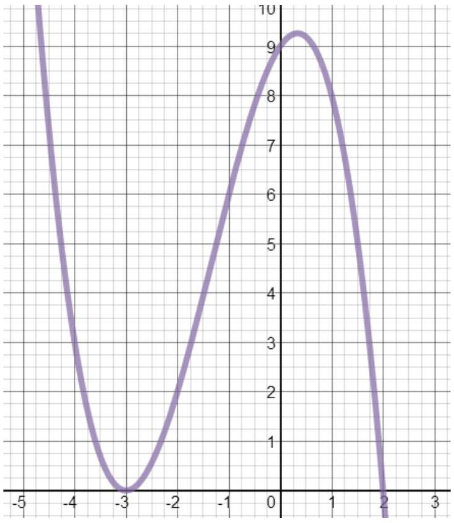
** The higher the even order number, the flatter the graph will be near the x-axis. **



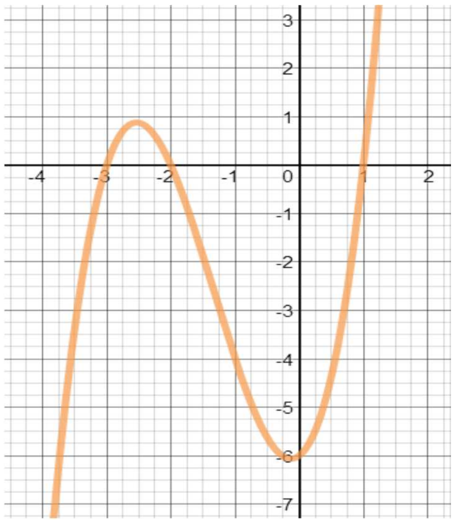
Example: For the following graphs of polynomial functions, determine

a) the least possible degree and sign of the leading coefficient
b) the x-intercepts and the factors of the function
c) the intervals where the function is positive and the intervals where it is negative
d) an equation for the polynomial function that corresponds to the graph

i)



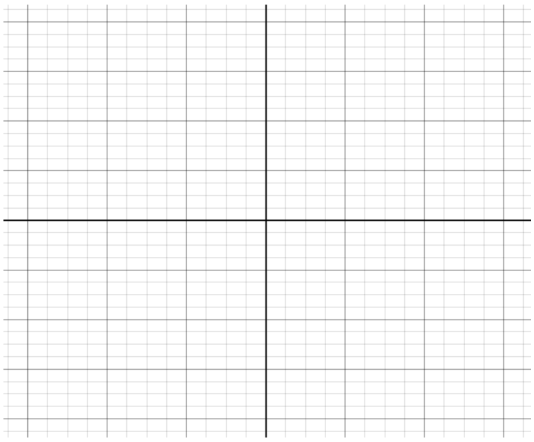
ii)



Example: Sketch a graph of the following polynomial functions defined by

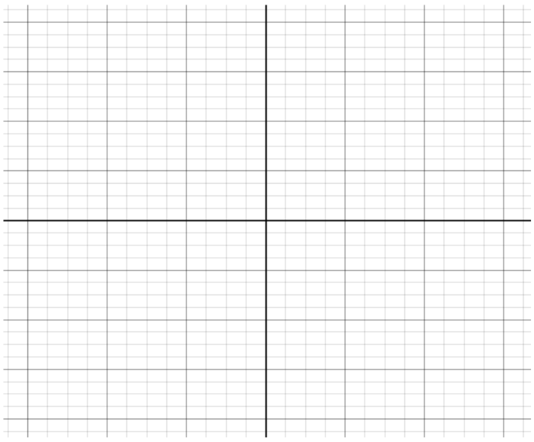
a) $y = -2(x + 1)^2(x - 2)$

Degree	Leading Coefficient	End Behaviour	Zeroes	y- intercept



b) $y = (x - 1)^2(x + 2)(x + 4)$

Degree	Leading Coefficient	End Behaviour	Zeroes	y- intercept

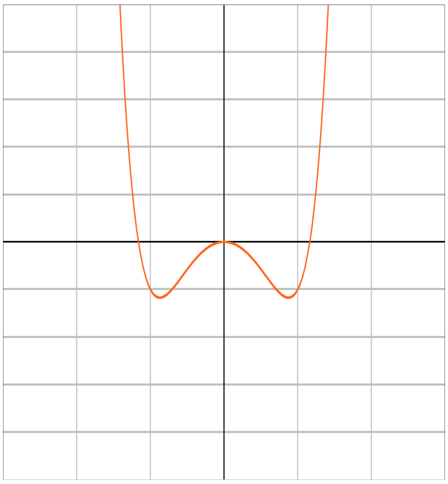


Even Functions

An even degree polynomial function is an "even function" if:

i) the exponent of each term is even.
ii) the function satisfies the property f(-x)= f(x) for all x in its domain
iii) the function is symmetric about the y-axis.

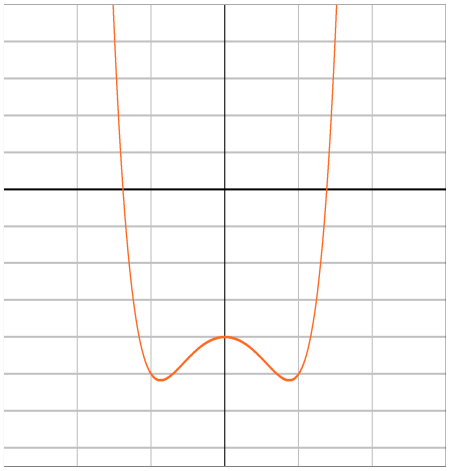
Example: $f(x) = 2x^6 + x^4 - 5x^2$



Even Function

Constant

Example: $f(x) = 2x^6 + x^4 - 5x^2 - 8$



Still Even Function

f(-x)= f(x) means that the reflection over the y-axis is the same as the original

*When a constant is added to an even function, the function remains even. The constant causes a vertical translation, and does not affect symmetry about the y-axis. **

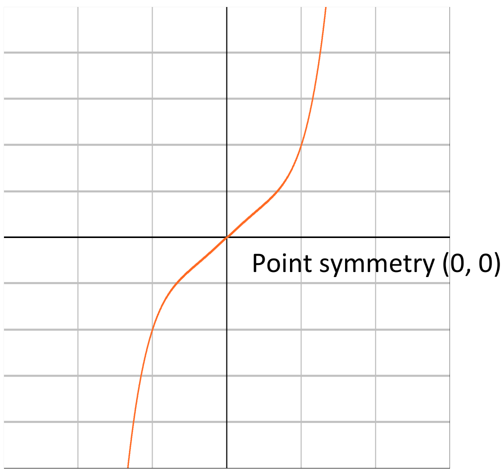
Odd Functions

An odd-degree polynomial function is an odd function if:

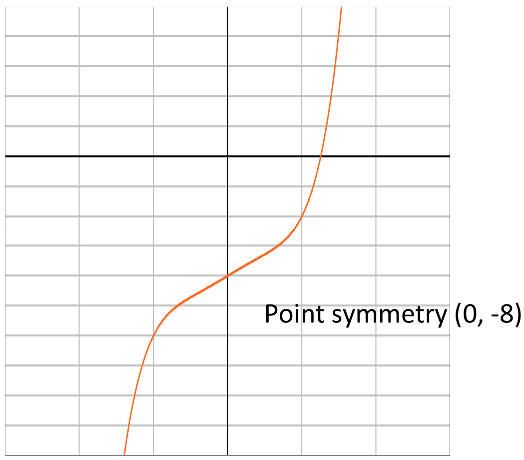
i) the exponent of each term is odd.
ii) the function satisfies the property f(-x) = - f(x) for all x in its domain
iii) the function is rotationally symmetric about the origin.

Example: $f(x) = 2x^5 - x^3 + 3x$

Example: $f(x) = 2x^5 - x^3 + 3x - 8$ ← Constant



Odd Function



No longer an Odd Function

f(-x)= - f(x) means that a reflection over the y-axis followed by a reflection over the x-axis is the same as the original (180° rotation about the origin)

*When a constant is added to an odd function, the function does not remain odd. The constant again causes a vertical translation, and does affect point symmetry about the origin. **

Example: Without graphing, determine if each polynomial function has line symmetry, point symmetry, or neither. Justify your answer.

a) $f(x) = -5x^4 + 3x^2 - 4$

b) $g(x) = x(2x + 3)(x - 2)$