Section 1.1 Power Functions

A **power function** is the simplest type of **polynomial function** and has the form $\mathbf{f(x) = ax^n}$, where **x** is a variable, **a** is a real number and **n** is a whole number.

A **polynomial expression** is an expression of the form:

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + . . . + a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0$$

Recall, we very seldom show exponent values of 1, and $x^0 = 1$

so $$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + . . . + a_3x^3 + a_2x^2 + a_1x + a_0$$

Where

- *n* is a whole number
- *x* is a variable
- the coefficients a_0, a_1, a_2, ... are real numbers
- the **degree** of the expression is *n*, the exponent on the greatest power of x
- a_n, is the coefficient of the greatest power of x, and is called **the leading coefficient**
- a_0, the term without a variable, is the **constant term**

A **polynomial function** has the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + . . . + a_3x^3 + a_2x^2 + a_1x + a_0$$

Traditionally, polynomial functions are written in descending order of powers of **x**.
(It keeps things looking nice and neat)

P.S the exponents in the function do not need to decrease consecutively, some terms may have zero as the coefficient. So $f(x) = 12x^4 + 2x^2 + 5$ is still a polynomial function, it just means that for ease we did not show the zero coefficient terms $f(x) = 12x^4 + 0x^3 + 2x^2 + 0x + 5$.

Some **Power Functions** have special names that are associated with their **degree**

Power Function	Degree	Name
y = a	0	Constant
y = ax	1	Linear
$y = ax^2$	2	Quadratic
$y = ax^3$	3	Cubic
$y = ax^4$	4	Quartic
$y = ax^5$	5	Quintic

Example: Determine which functions are polynomials. Justify your answer.
State the degree and the leading coefficient of each polynomial function.

a) $g(x) = \cos x$

b) $f(x) = 3x^4$

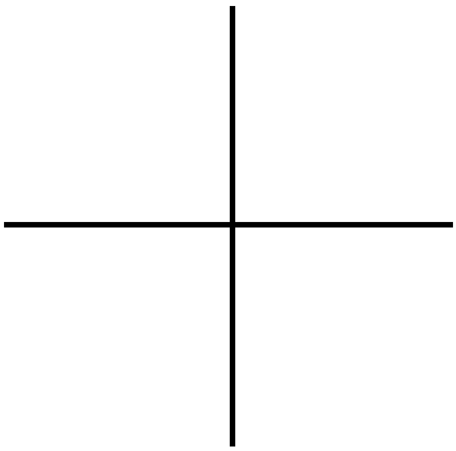
c) $f(x) = x^5 - 3x^3 + 7x^2 - x + 1$

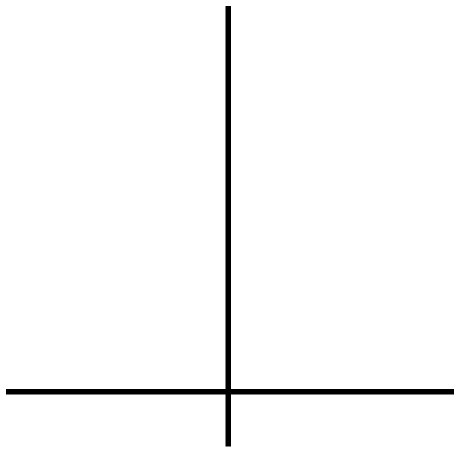
d) $h(x) = 4^x$

Investigate Power Functions

Graph the following using available technology. Make a sketch with labels.

$y = x$, $y = x^3$, $y = x^5$, $y = x^7$ and $y = x^2$, $y = x^4$, $y = x^6$, $y = x^8$





Power functions have similar characteristics depending on whether their degree is even or odd.

Odd Degree Power Functions: Graphs that curve from quadrant 3 to quadrant 1. The higher the exponent the closer the curve gets to the y-axis.

Even Degree Power Functions: Graphs that make a U-shape. The higher the exponent the U shape gets closer to the y-axis.

End behaviour: The end behaviour of a function is the behaviour of the **y-values** as x increases (that is, as x approaches positive infinity, $x \to \infty$) and as x decreases (that is, as x approaches negative infinity, $x \to -\infty$)

Example: Write each of the following power functions in the appropriate row of the second column of the table below. Give reasons for your choices.

$y = 2x$ $y = 5x^6$ $y = -3x^2$ $y = x^7$ $y = -\frac{2}{5}x^9$

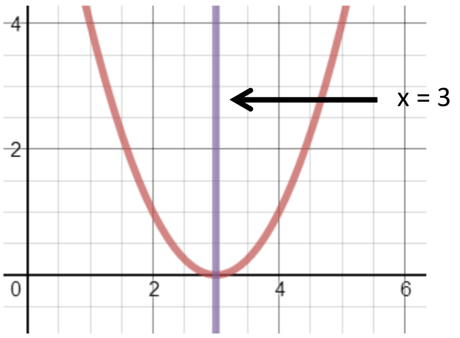
$y = -4x^5$ $y = x^{10}$ $y = -0.5x^8$

End Behaviour	Function	Reasons
Extends from quad 3 to quad 1		
Extends from quad 2 to quad 4		
Extends from quad 2 to quad 1		
Extends from quad 3 to quad 4		

Line Symmetry

A graph has line symmetry if there is a line **x = a** that divides the graph into two equal parts such that one part is a reflection of the other in the line **x = a**.

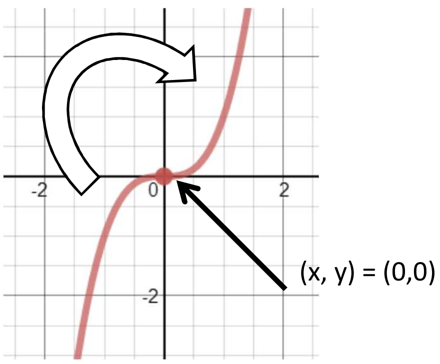
- Even-degree power functions have line symmetry.



Point Symmetry

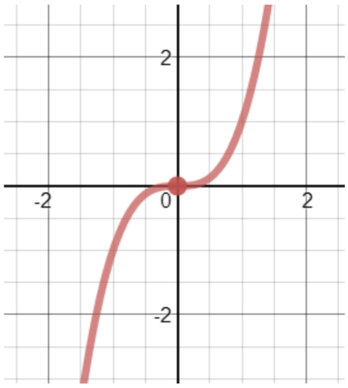
A graph has point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b).

- Odd-degree power functions have a point of symmetry.

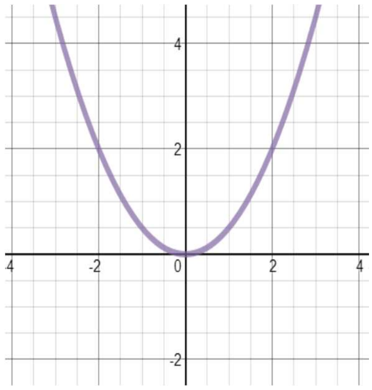


Example: For each of the following functions, state the domain and range, describe the end behaviour and identify any symmetry.

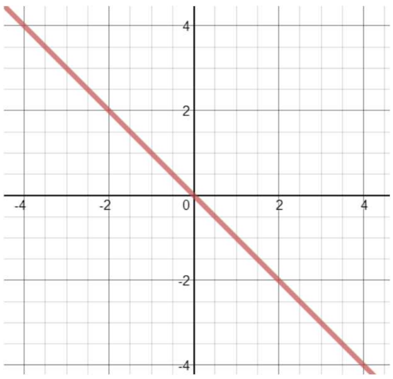
a)



b)



c)



Example: The volume of a helium balloon is given by the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius of the balloon, in meters and $r \in [0, 5]$

a) Sketch V(r).

b) State the domain and range in this situation.

c) Describe the similarities and difference between the graph of V(r) and the graph of $f(x) = x^3$