MHF4U Unit 1 Polynomial Functions

Section	Pages	Questions
Prereq Skills	2 – 3	# 1ace, 2cde, 3bce, 4, 5, 6, 7, 8ace, 9, 10b, 11b, 12 & Factoring Practice
1.1	11 – 14	# 1, 2, 3, 4, 5, 7, 8, 9(in class)
1.2	26 – 29	# 1, 2, 3, 4abcf, 5, 6, 7, 8, 11(in class), 12
1.3	39 – 41	# 1bc, 2ab, 3, 5, 6ac, 7bd, 9(don't graph), 11a, 12ab, 14*
1.4	49 – 52	# 1acd, 2, 3, 4, 5, 6, 7abc, 8ac, 9, 10, 12, 14*
1.5	62 – 64	# 1, 2, 3, 4, 5, 7a(don't graph)bcd, 10ab
1.6	71 – 73	# 1, 2, 3, 4, 5, 7, 9, 10*, 11*
Review	74 – 77 79	# 1-11, 12(don't graph), 13, 14, 15, 17, 18 # 8abd, 13

Note: Questions with an asterisk* are extra questions that are optional for the daily homework. However, they are potential "extended-type" questions that could be included on a unit test.

Unit 1 - Lesson 1

Prerequisite Skills

Grade 9

- Slope
- equation of a straight line
- x intercept
- y intercept
- first differences

Grade 10

- factoring
- quadratic equation in vertex form
- second differences
- basic transformations of quadratic
- distance between two points in space

Grade 11

- function notation
- transformations of functions
- domain and range
- vertical asymptotes and holes
- radicals and rational functions
- sketching functions

Function Notation

To represent functions, we use notations such as f(x) and g(x).
ex. Linear function: $y = 2x + 1$

In Function notation: $f(x) = 2x + 1$

The notation f(x) is read "f of x" or "f at x".
It means that the expression that follows contains x as a variable.

For example: f(3) means substitute 3 for every x in the expression and solve for y, or f(3).

ex. Quadratic function: $y = x^2 – 4x + 7$
$f(3) = (3)^2 -4(3) + 7$
$f(3) = 9 – 12 + 7$
$f(3) = 4$

Therefore when $x = 3$, $y = 4$ or $f(3) = 4$

For example: f(2a) means substitute 2a for every x in the expression and solve for y, or f(2a).

ex. Linear function: $y = 3x – 10$ find f(2a)
$f(2a) = 3(2a) – 10$
$f(2a) = 6a – 10$

Therefore when $x = 2a$, $y = 6a – 10$ or $f(2a) = 6a – 10$
we can create new equations or functions

Examples: Determine each value for the function $f(x) = x^2 – 4x + 1$

a) f(0) b) f(-2) c) f(1/2)

d) f(3x) e) -2f(2x)

Slope and y-intercept of a line

The equation of a line, written in the form y = mx + b has m = slope and b = y-intercept

Examples: Determine the slope and y-intercept of the following lines.

a) y = 3x -1	b) 2x - 7y = 14	c) y + 2 = 7(x - 1)
m = 3	7y = 2x – 14	y + 2 = 7x - 7
b = - 1	y = 2/7x – 2	y = 7x – 9
	m = 2/7	m = 7
	b = - 2	b = - 9

Equation of a Line (y = mx + b)

To write the equation of a line, you need the slope and the y-intercept
Recall: The Slope Formula
Given two points (x_1, y_1) and (x_2, y_2), the slope is given by: $m = \frac{\Delta y}{\Delta x}, m = \frac{y_2 - y_1}{x_2 - x_1}$

Examples: Determine the equation of the line that satisfies each set of conditions.

a) Slope is -1 and the y-intercept is 7.

y = -1x + 7

b) Slope is 2 and it passes through the point (1, -4).

y = 2x + b	or	y = m(x – p) + q
-4 = 2(1) + b		y = 2(x – 1) - 4
-4 = 2 + b		y = 2x – 2 – 4
-6 = b		y = 2x – 6
y = 2x – 6		

c) Line passes through the points (-2, 0) and (2, 4).

$m = \frac{4-0}{2-(-2)} = \frac{4}{4} = 1$

y = m(x – p) + q

y = 1(x – 2) + 4

y = 1x – 2 + 4

y = 1x + 2

Finite Differences

Finite differences can be used to determine whether a function is linear, quadratic or neither.
Finite differences can ONLY be used if the x-values in the table are increasing/decreasing by the same amount.
If the 1st differences are constant, the function is linear.
If the 2nd differences are constant, the function is quadratic.

Ex: Use finite differences to determine whether the functions below are linear, quadratic, or neither.

a)

X	Y	1st Diff	2nd Diff
1	4		
2	1		
3	0		
4	1		
5	4		

b)

X	Y	1st Diff	2nd Diff
-2	9		
-1	7		
0	5		
1	3		
2	1		

Interval Notation:

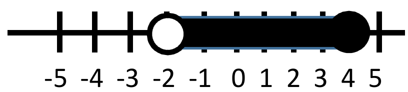
- Used to express a set of numbers
- Intervals that are infinite are ∞ and -∞
- Square brackets indicate the end value is included, round brackets indicate the end value is not included
- A round bracket is always used with the ∞ symbol

Sets of real numbers may be expressed in a number of ways.

a) Inequality b) Interval Notation c) Graphically (number line)

Ex.

$-2 < x \leq 4$ (-2, 4]

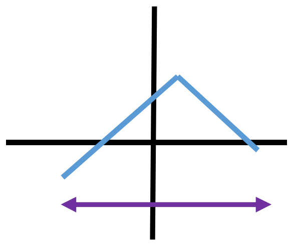


Example: All possible intervals for real numbers a and b, where a < b:

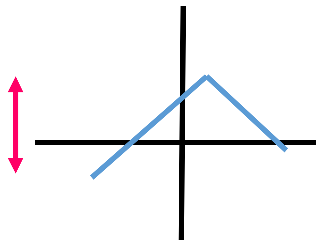
Bracket Interval	Inequality	Number Line	In Words
			The set of all real numbers x such that
(a, b)	$a < x < b$		x is greater than a and less than b
$(a, b]$	$a < x \leq b$		x is greater than a and less than or equal to b
$[a, b)$	$a \leq x < b$		x is greater than or equal to a and less than b
$[a, b]$	$a \leq x \leq b$		x is greater than or equal to a and less than or equal to b
$[a, \infty)$	$x \geq a$		x is greater than or equal to a
$(-\infty, a]$	$x \leq a$		x is less than or equal to a
(a, ∞)	$x > a$		x is greater than a
$(-\infty, a)$	$x < a$		x is less than a
$(-\infty, \infty)$	$-\infty < x < \infty$		x is an element of the real numbers

Domain and Range

The domain of a function is the **set of all first coordinates (x-values)** of the relation.
The range of a function is the **set of all second coordinates (y-values)** of the function.



Domain

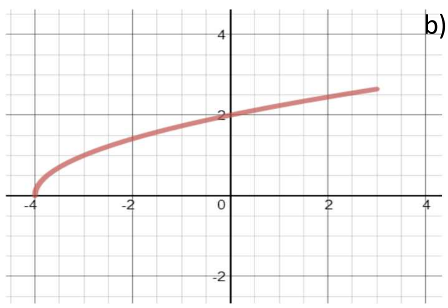


Range

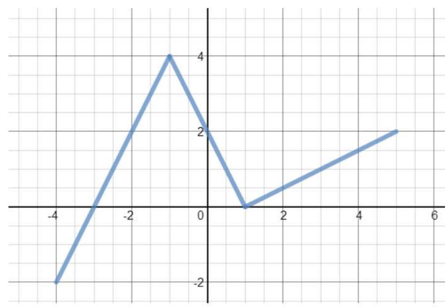
Examples:

1. Given the following relations, state the domain and range.

a)

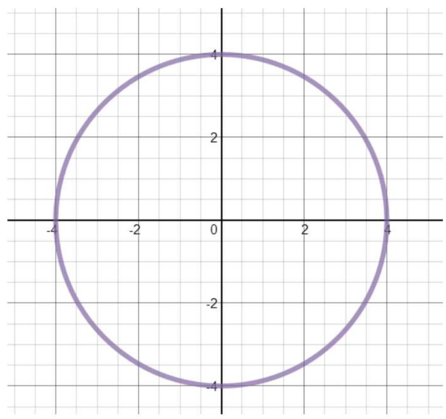


b)

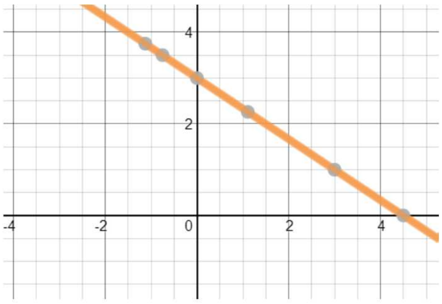


c) {(1, 2), (3, 4), (4, 6), (7, 10)}

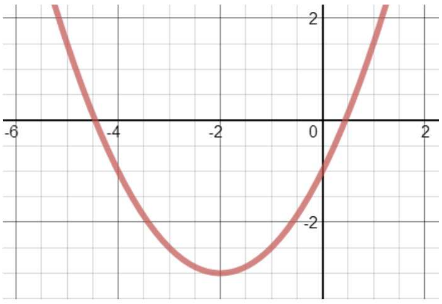
d)



e)



f)



2. Given the equation of the following functions, sketch each function and state their domain and range.

a) $y = x - 5$

b) $y = x^2 + 7$

c) $y = -2(x + 4)^2 + 3$

d) $y = \sqrt{x - 3}$

e) $y = \frac{1}{x+3}$

Quadratic Functions

There are 3 forms used to model quadratic functions.

Form	Model	Properties	Example
Standard Form	$y = ax^2 + bx + c$ where a, b, & c are constants and $a \neq 0$	• If $a > 0$, the parabola opens up and has a minimum. • If $a < 0$, the parabola opens down and has a maximum • c is the y-intercept	$y = 3x^2 - 4x + 7$ a = 3 and 3 >0, so the parabola opens up and has a minimum. 7 is the y-intercept.
Factored Form	$y = a(x – r)(x – s)$ where a, r & s are constants and $a \neq 0$	• If $a > 0$, the parabola opens up and has a minimum. • If $a < 0$, the parabola opens down and has a maximum • Values for r and s are used to find the x-intercepts or zeros.	$y = -2(x + 4)(x – 3)$ a = -2 and -2 < 0, so the parabola opens down and has a maximum. The x-intercepts are at -4 and 3.
Vertex Form	$y = a(x – p)^2 + q$ where a, p & q are constants and $a \neq 0$	• If $a > 0$, the parabola opens up and has a minimum. • If $a < 0$, the parabola opens down and has a maximum • (p, q) is the vertex	$y = 0.5(x – 3)^2 + 5$ a = 0.5 and 0.5 > 0, so the parabola opens up and has a minimum. The vertex is at (3,5).

Examples: Determine the equation of a quadratic function that satisfies each set of conditions.

a) x-intercepts at -2 and -6, y-intercept at 24.

b) x-intercept at -1, passing through the point (-2, 6).

c) Vertex at (-4, 7) and passing through the point (1, 12).

Factoring Polynomials

Always look for a greatest common factor (GCF) first!

Ex. $8x^3 + 6x^2 = 2x^2(4x + 3)$

If the expression is a binomial, look for a Difference of Squares.

Ex. $x^2 – 25 = (x – 5)(x + 5)$

If the expression is a trinomial in the form $x^2 + bx + c$, look for the Sum (b) and Product (c).

Ex. 1. $x^2 + 9x + 20 = (x + 4)(x + 5)$

Add 9	Mult 20
7	1, 20
-7	2, 10
5	4, 5

If the expression is a trinomial in the form $ax^2 + bx + c$, look for the Sum (b) and Product (a x c), use decomposition approach.

Ex. 2. $2x^2 – 5x + 3 = 2x^2 – 2x – 3x + 3$
$2x(x – 1) – 3(x – 1)$
$(x – 1) (2x – 3)$

Add -5	Mult 6
7	1, 6
-7	-1, -6
5	2, 3
-5	-2, -3

Remember to factor fully where possible.

Ex. 3. $3x^2 – 48 = 3(x^2 – 16)$
$3(x – 4)(x + 4)$

Ex. 4. $2x^3 – 14x^2 + 24x = 2x(x^2 –7x + 12)$
$2x(x – 4)(x – 3)$

Examples: Factor Fully

a) $3a^4b^2 -6a^2b^3 + 12ab^4$

b) $36x^2 – 49$

c) $9a^2 – 1$

d) $x^2 -5x -14$

e) $6a^2 – 9a – 6$

f) $10y^3 +5y^2 -5y^3$

g) $6x^2 -22x – 40$

h) $4a^2 – 25b^2$

Determining x-intercepts or roots of quadratic functions.

Standard form – factor if possible, set the factors equal to zero.

Ex. $y = x^2 + 10x + 21 = 0$
$y = (x + 3)(x + 7) = 0$

$x = -3$, or $x = -7$

Standard form – factor is not possible, use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (quadratic equation)

Ex. $2x^2 - 4x - 10$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-10)}}{2(2)}$$
$$x = \frac{4 \pm \sqrt{16 + 80}}{4}$$
$$x = \frac{4 \pm \sqrt{96}}{4}$$

$x = 3.45$ or $x = -1.45$

Vertex form – set equation equal to zero, isolate x.

Examples: Determine the x-intercepts, the vertex, the direction of opening, and the domain and range of each quadratic function. Sketch a graph of the function.

a) $y = 2(x - 2)(x + 5)$

b) $y = 3(x - 5)^2 - 9$

c) $y = -3x^2 + 17x + 6$

d) $y = 2x^2 - 12x + 7$

Transformations   y = af(k(x-d)) + c

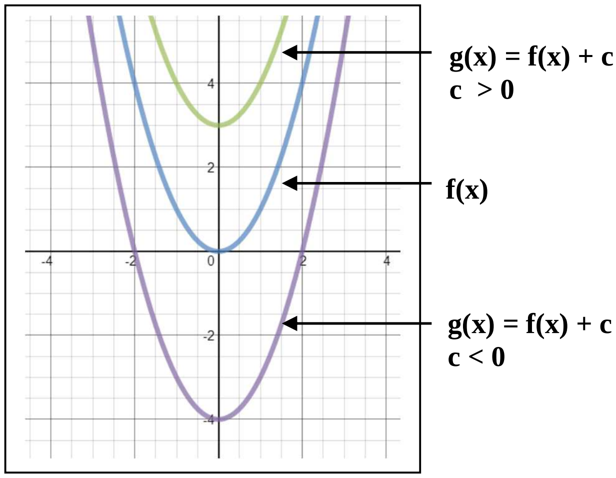
Translations

A transformation that results in a shift of the original figure without changing its shape.

Vertical Translation of **c** units:

The graph of the function **g(x) = f(x) + c**

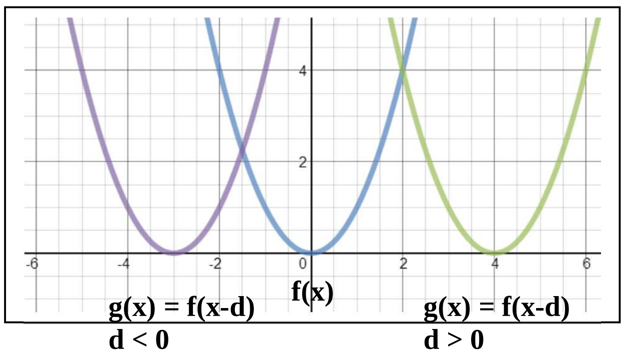
- when c is positive, the translation is UP by c units.
- when c is negative, the translation is DOWN by c units.



Horizontal Translation of **d** units:

The graph of the function **g(x) = f(x - d)**

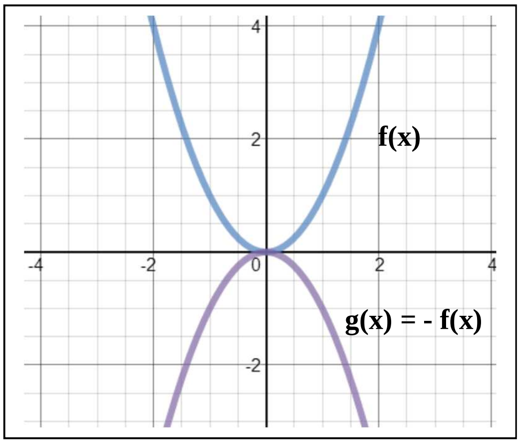
- when d > 0, the translation is to the RIGHT by d units.
- when d < 0, the translation is to the LEFT by d units.



Reflections
A transformation in which a figure is reflected over a reflection line.

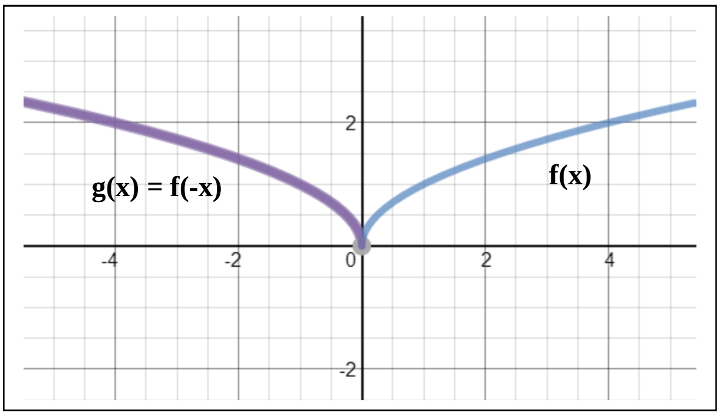
Reflection in the X-axis/Vertical Reflection:

The graph of g(x) = -f(x)



Reflection in the Y-axis/Horizontal Reflection:

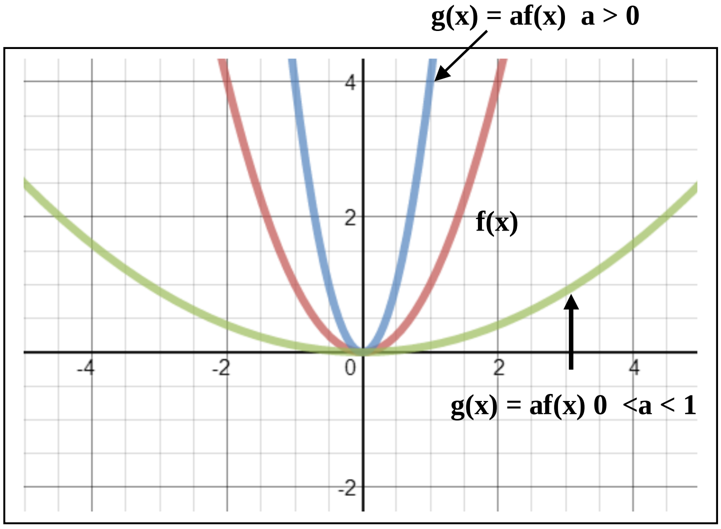
The graph of g(x) = f(-x)



Vertical Stretch and Compression:

The graph of the function g(x) = af(x), a>0

- when |a|>1, there is a VERTICAL STRETCH by a factor of a.
- when 0<|a|<1, there is a VERTICAL COMPRESSION by a factor of a.
- Points on the x-axis are invariant.

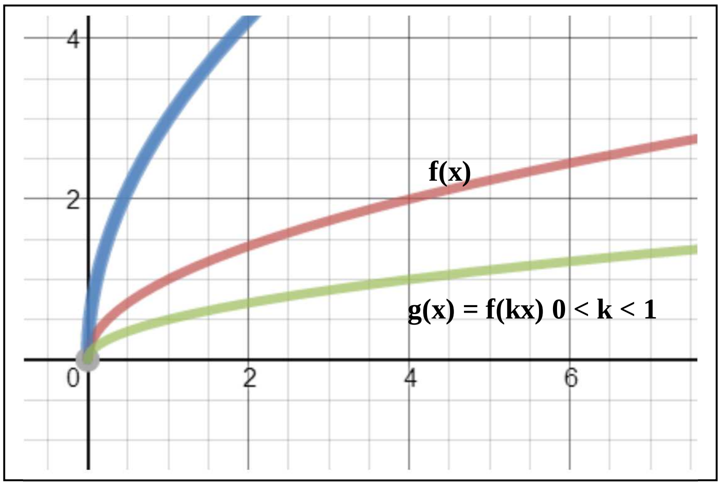


Horizontal Stretch and Compression:

The graph of the function g(x) = f(kx), k > 0

- when |1/k| >1, there is an EXPANSION (STRETCH) by a factor of 1/k.
- when 0 < |k| <1, there is a COMPRESSION by a factor of 1/k.

g(x) = f(kx) k > 0



Examples: Identify each transformation of the function y = f(x).

a) y = 2f(x) + 1

b) $y = -\frac{1}{3}f(x-2)$

c) y = f(-3x)

d) y = -2 f(3x + 3) - 4

Examples: Write an equation for the transformed function of each base function. State the domain and range of each.

a) f(x) = x^2, is reflected in the x-axis, stretched vertically by a factor of 3, translated to the left 6 units and down 5 units.

b) $f(x) = \sqrt{x}$, is compressed horizontally by a factor of 0.5, stretched vertically by a factor of 3, reflected in the y-axis, and translated right 4 units.