MHF4U Unit 5 Trigonometry

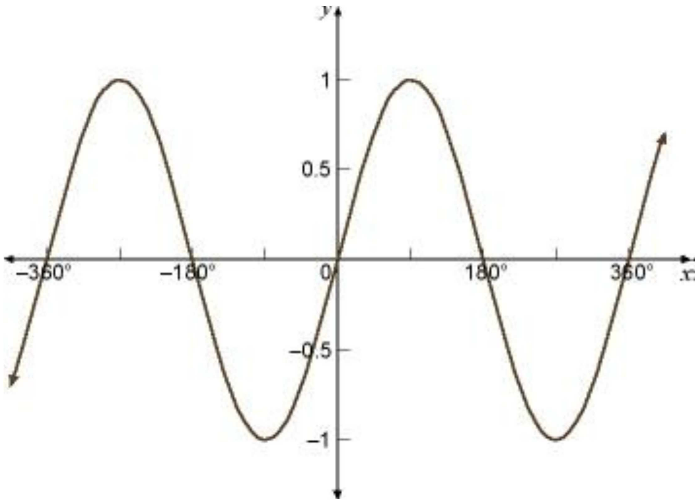
Section	Pages	Questions
Prereq Skills	250 - 251	#1ac, 2ac, 3ace, 4ace, 5, 6, 8(graph the function), 9(graph the function), 10, 11, 12ab, 15
5.1	258 - 259	#1ad, 2ad, 3ad, 4ad, 5bc, 6bc, 7ab, 8ab, 9, 10, 11, 12, 17
5.2	275 - 279	#1abce, 2, 3, 4, 5(don't graph), 6(don't graph), 8ab, 9ab, 10, 11, 12a, 13a, 19a
5.3	267 - 269	#7, 9a, 10, 13, 15*, 16*, 18
5.4	287 - 289	#1abe, 3ab, 5be, 7cd, 9, 10, 11, 12, 13bc, 14, 16, 17, 18, 19, 20, 22*, 26
5.5	296 - 299	#1, 3, 4abef, 6, 10, 11ab, 12
Review	300 – 301 302 - 303	#1, 2, 3, 4, 6a, 7, 8, 9, 10, 11, 12 #2-7, 9-15

Note: Questions with an asterisk* are extra questions that are optional for the daily homework. However, they are potential "extended-type" questions that could be included on a unit test.

Section 5.0 Prerequisite Skills

Graph of $y = \sin x \ for \ -360° \leq x \leq 360°$.

x	y
-360°	0
-270°	1
-180°	0
-90°	-1
0°	0
90°	1
180°	0
270°	-1
360°	0



Key Features

Period = 360°,

Max for y = 1,

Min for y = -1

Amplitude = 1,

Phase Shift = 0°

Vertical Displacement = 0

x-intercepts: -360°, -180°, 0°, 180°, 360°

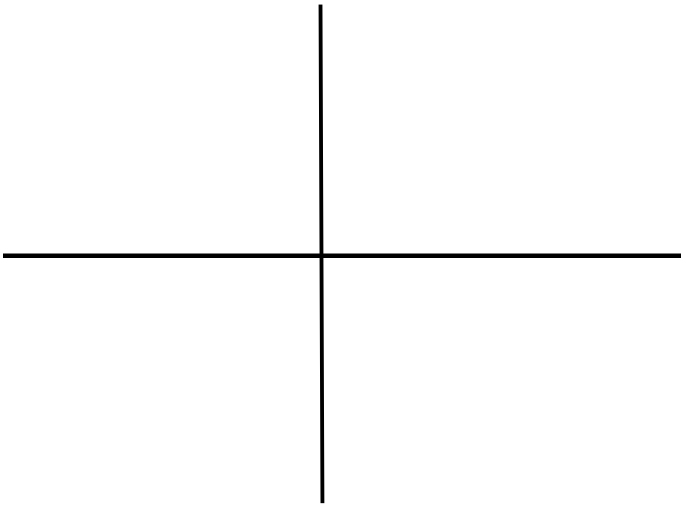
y-intercept: 0

Domain: $\{x \in R | -360° \leq x \leq 360°\}$

Range: $\{y \in R | -1 \leq y \leq 1\}$

Graph of y $= \cos x$ for $-360° \leq$ x $\leq 360°$.

x	y
-360°	
-270°	
-180°	
-90°	
0°	
90°	
180°	
270°	
360°	



Period =

Max for y =

Min for y =

Amplitude =

Phase Shift =

Vertical Displacement =

x-intercepts:

y-intercept:

Domain:

Range:

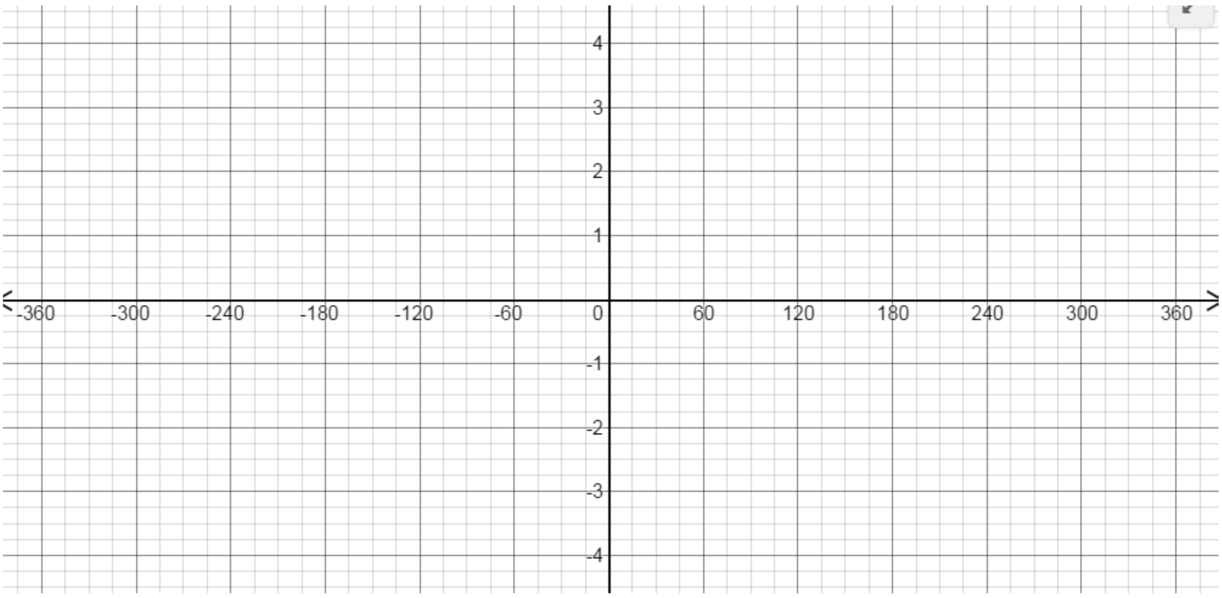
Transformations of Sinusoidal Functions

$y = a\, sin\big(k(\theta - d)\big) + c$ $\qquad\qquad$ $y = a\, cos(k(\theta - d)) + c$

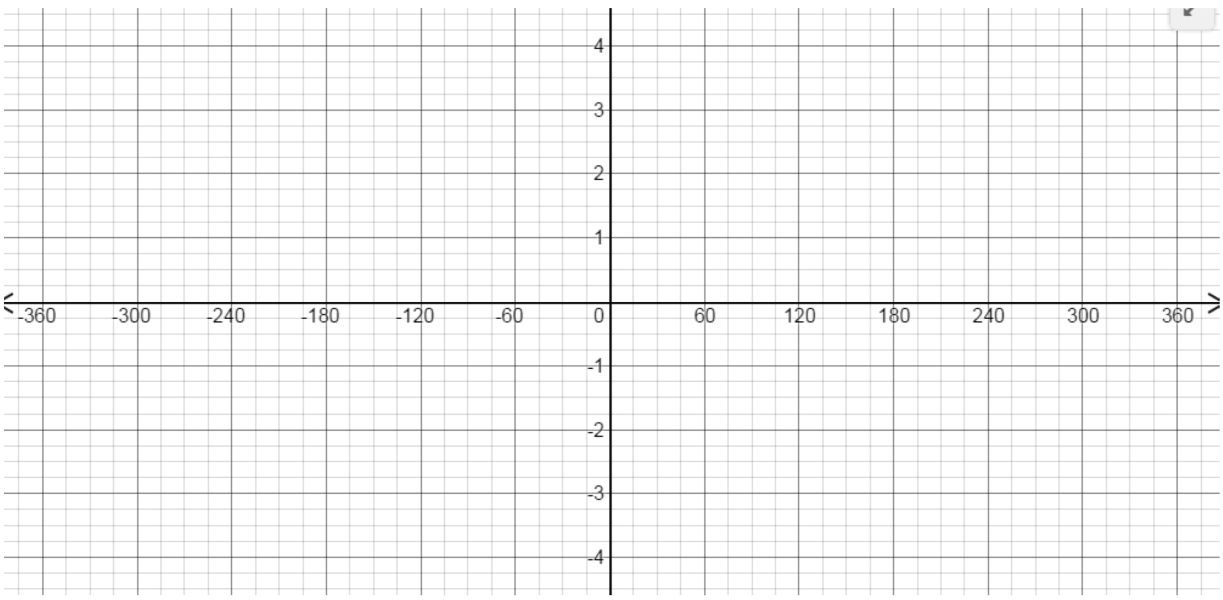
a = amplitude (vertical stretch or compression)
k = horizontal stretch or compression
d = phase shift (horizontal translation)
c = vertical displacement (vertical translation)

Period = $360^{o}/\ k$ (time needed to complete one cycle or wavelength)
Frequency = 1 / Period (number of cycles per unit of time)

Example: Describe the transformations that must be applied to the graph of $f(x) = \sin x$ to obtain the graph $g(x) = -0.5sin[3(x + 150°)] + 3$. Graph the functions $f(x)$ and $g(x)$ on the same grid and state the domain and range.

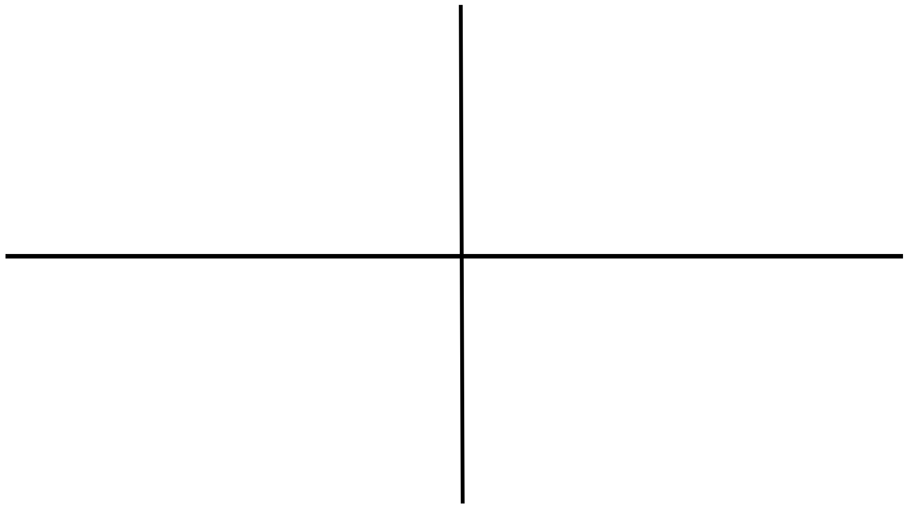


Example: Describe the transformations that must be applied to the graph of $f(x) = \cos x$ to obtain the graph $g(x) = 2cos[2(x - 120°)] - 2$. Graph the functions $f(x)$ and $g(x)$ and state the domain and range.



Graph of $y = \tan x$ for $-360° \leq x \leq 360°$.

x	y
-360°	
-315°	
-270°	
-225°	
-180°	
-135°	
-90°	
-45°	
0°	
45°	
90°	
135°	
180°	
225°	
270°	
315°	
360°	



<u>Key Features</u>

Period =

Max for y =

Min for y =

Amplitude =

Phase Shift =

Vertical Displacement =

x-intercepts:

y-intercept:

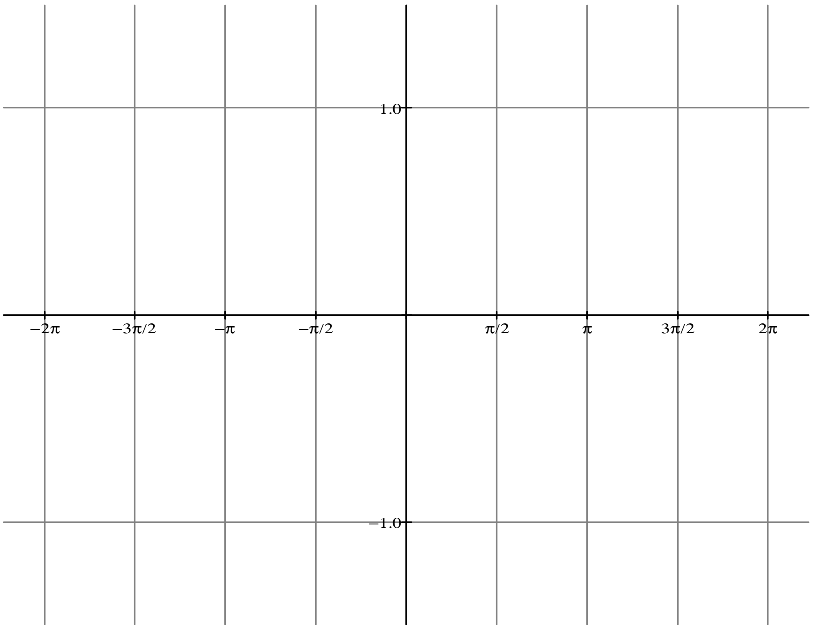
Domain:

Range:

Section 5.1 **Graphs of Sine, Cosine, and Tangent Functions**

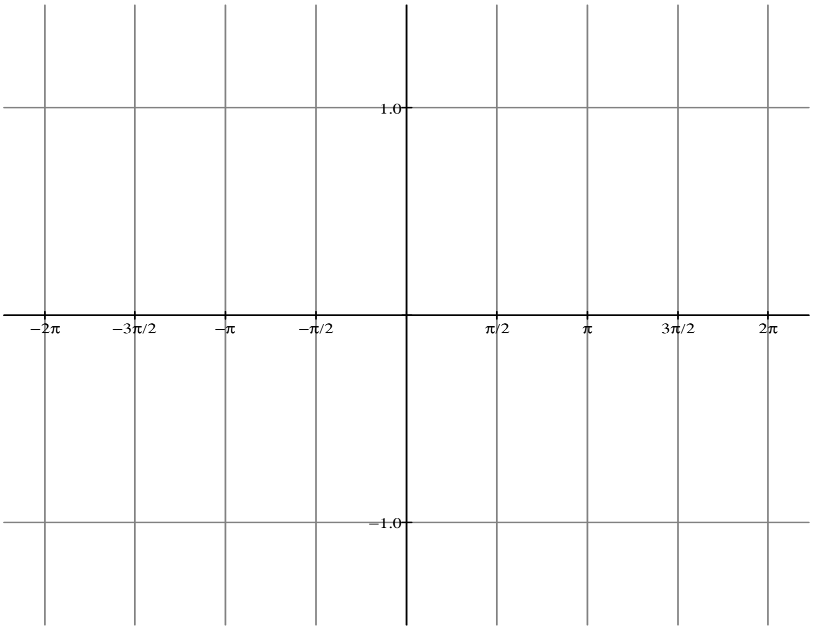
Graph of $y = sin\theta \, for \, -2\pi \leq \theta \leq 2\pi$

θ	θ	y
-360°	-2π	
-270°	-3π/2	
-180°	-π	
-90°	-π/2	
0°	0	
90°	π/2	
180°	π	
270°	3π/2	
360°	2π	



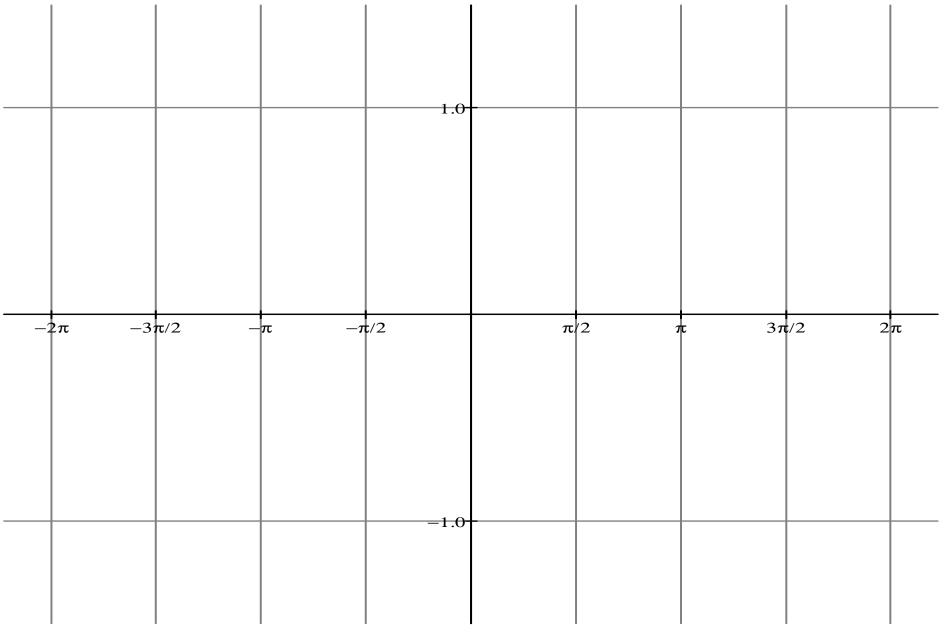
Graph of y $= \cos\theta$ for $-2\pi \leq \theta \leq 2\pi$.

θ	y



Graph of y $= \tan\theta$ for $-2\pi \le x \le 2\pi$.

θ	y



Basic Transformations of $y = \sin\theta$ and $y = \cos\theta$ still hold true for angles measured in radians.

Amplitude $\Rightarrow y = a\sin\theta \ and\ y = a\cos\theta$

- "a" is the amplitude of the function.
- When "a" is negative, a reflection exists over the x-axis.
- a = (maximum- minimum) / 2

Vertical Displacement $\Rightarrow y = \sin\theta + c \ and\ y = \cos\theta + c$

- The function moves up or down along the y-axis by "c" units.
- c = (maximum + minimum) / 2

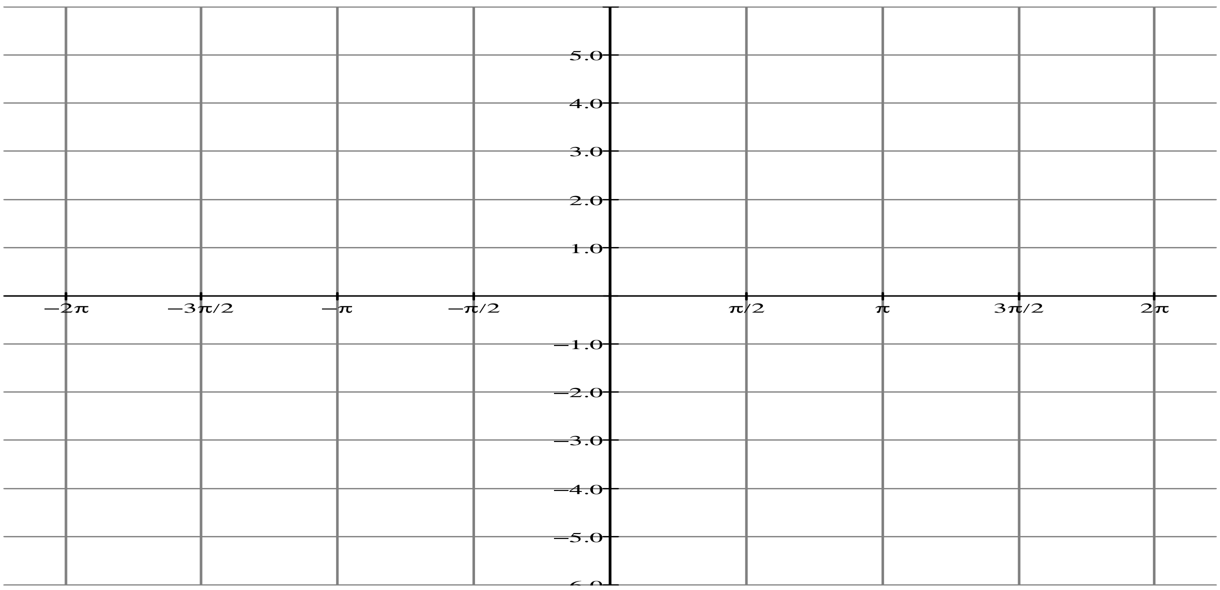
Phase Shift $\Rightarrow y = \sin(\theta - d) \ and\ y = \cos(\theta - d)$

- The function moves left or right along the θ-axis by "d" units.

Period Change $\Rightarrow y = \sin k\theta \ and\ y = \cos k\theta$

- The function has a new period given by p = 2π / k.
- So k = 2π / p

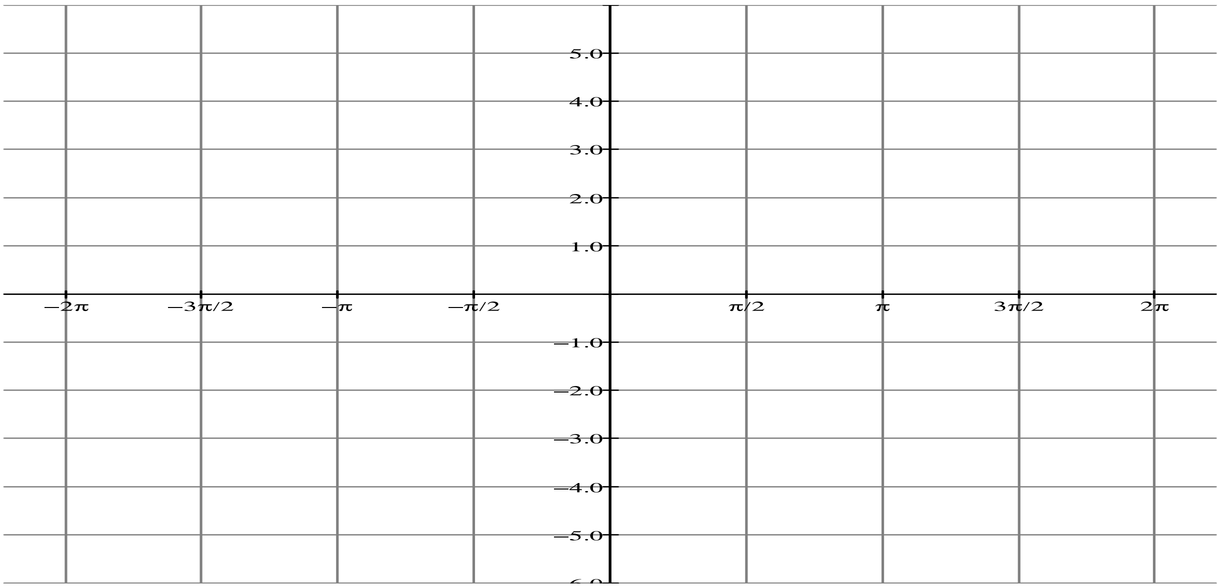
Example: Transform the graph of $y = \sin x$ to obtain $y = \sin x - 2$, over two cycles.



Amplitude: Period: Phase Shift:

Domain: Range: Vert.Displacement:

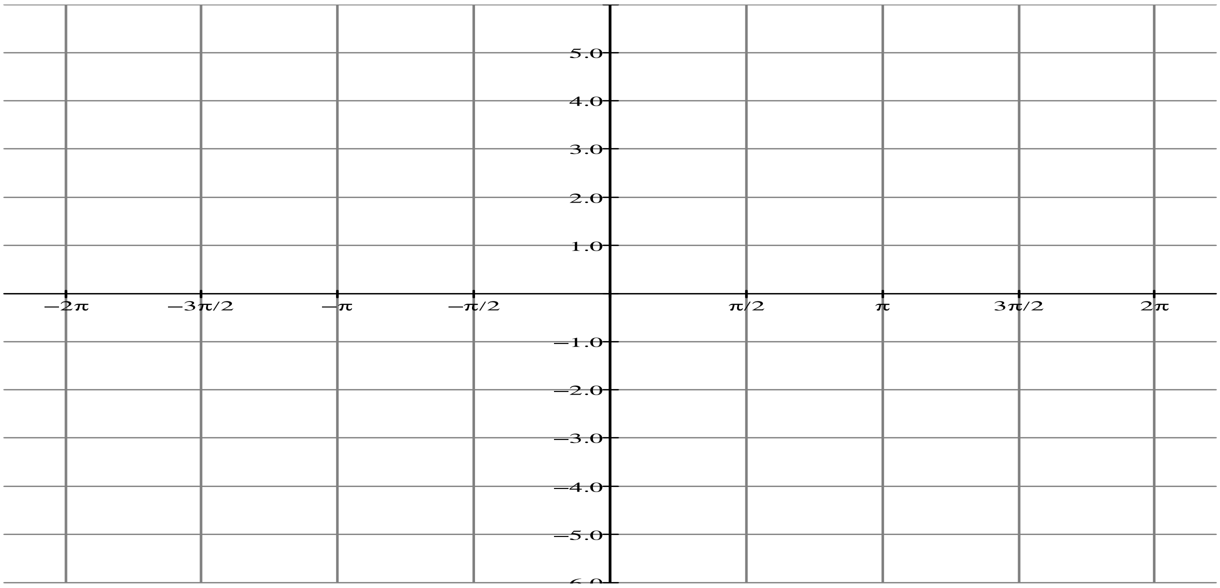
Example: Transform the graph of $y = \cos x$ to obtain $y = -3\cos x$, over two cycles.



Amplitude: Period: Phase Shift:

Domain: Range: Vert.Displacement:

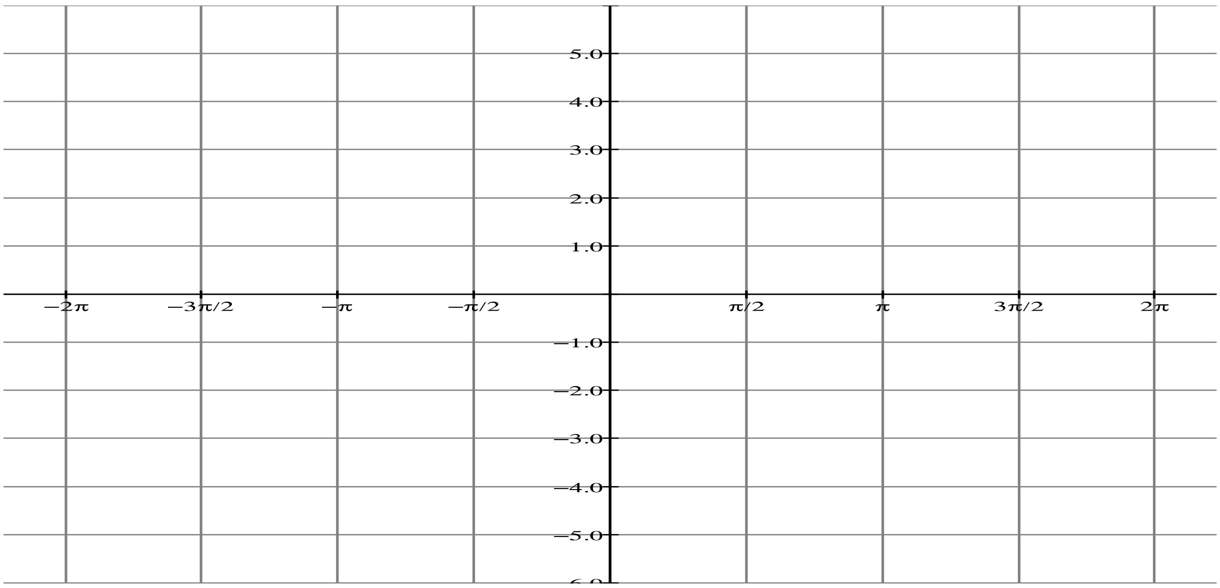
Example: Transform the graph of $y = \sin x$ to obtain $y = \sin\left(x - \frac{\pi}{3}\right)$, over two cycles.



Amplitude: Period: Phase Shift:

Domain: Range: Vert.Displacement:

Example: Transform the graph of $y = \cos x$ to obtain $y = \cos 2x$, over two cycles.



Amplitude: Period: Phase Shift:

Domain: Range: Vert.Displacement:

Example: A cosine function has a period of 6π, a maximum value of 5, and a minimum value of -9. Assuming there is no phase shift, determine an equation representing this cosine function in the form $y = acos(kx) + c$.

Example: One cycle of a sine function begins at $x = -\pi/4$ and ends at $x = 5\pi/4$.

a) Determine the period of the function.

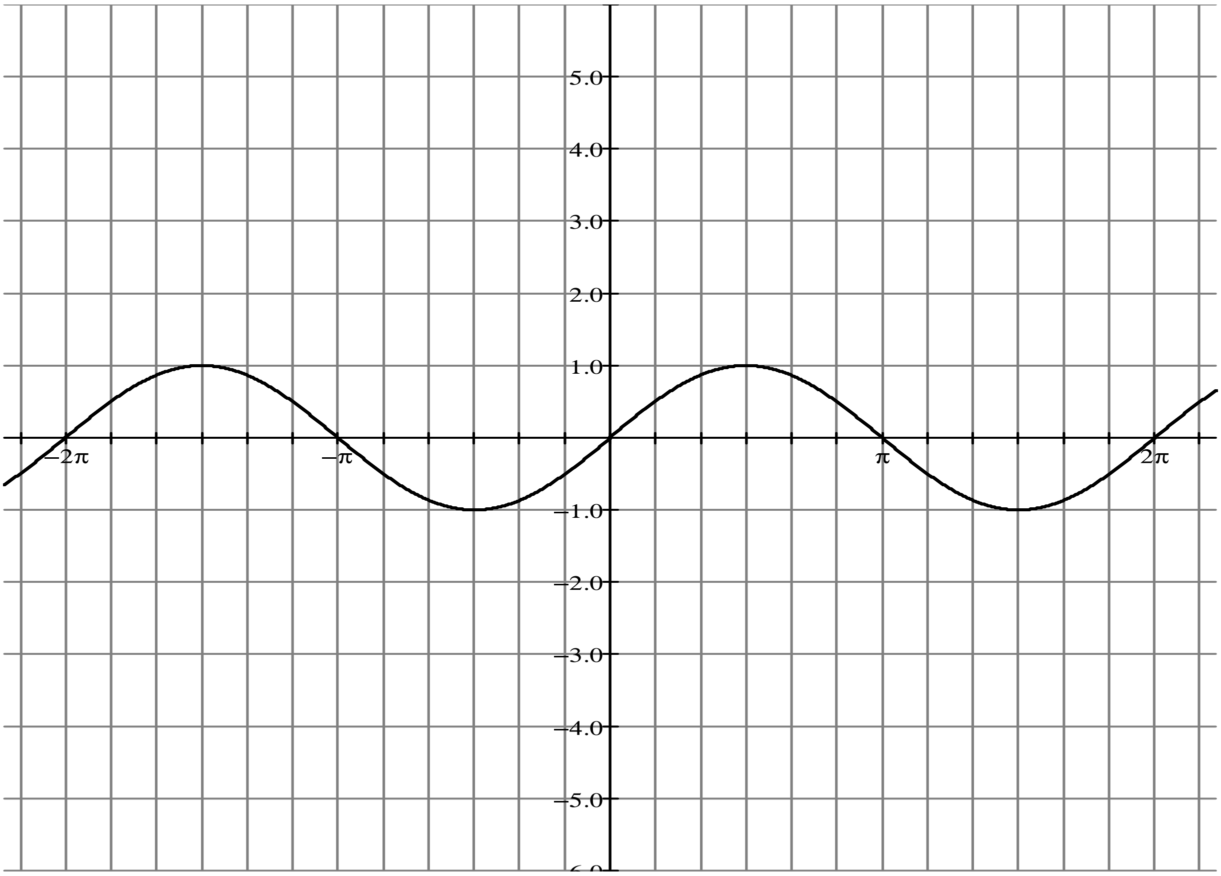
b) Determine the phase shift of the function.

c) Write the equation of the function in the form $y = sin[k(x - d)]$

Section 5.2 **Graphs of Reciprocal Trigonometric Functions**

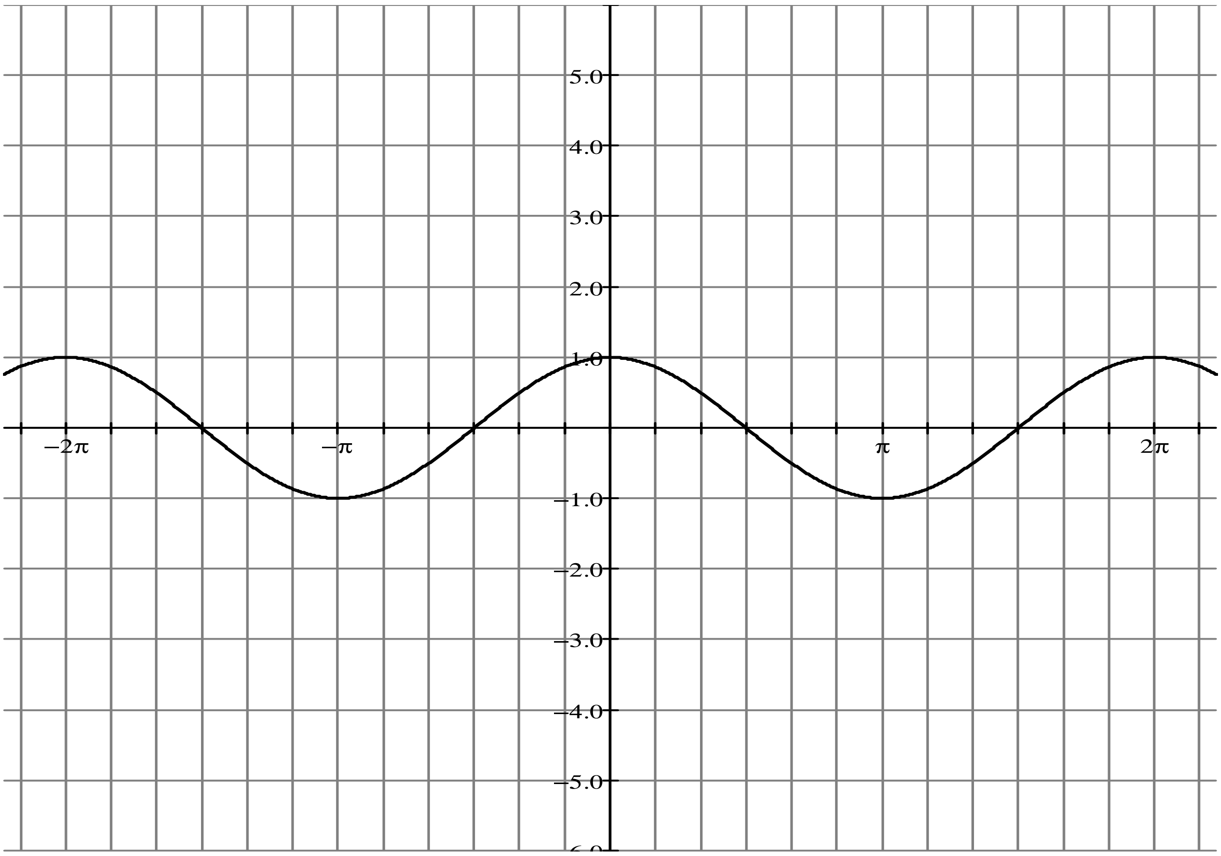
Below is the graph of $y = \sin x$. Recalling that $\csc x = 1/\sin x$, sketch the graph of $y = \csc x$ in the interval $x \in [-2\pi, 2\pi]$.

x	sin x	csc x
0	0	
π / 6	0.5	
π / 3	0.866	
π / 2	1	
2π / 3	0.866	
5π / 6	0.5	
π	0	
7π / 6	-0.5	
4π / 3	-0.866	
3π / 2	-1	
5π / 3	-0.886	
11π / 6	-0.5	
2π	0	



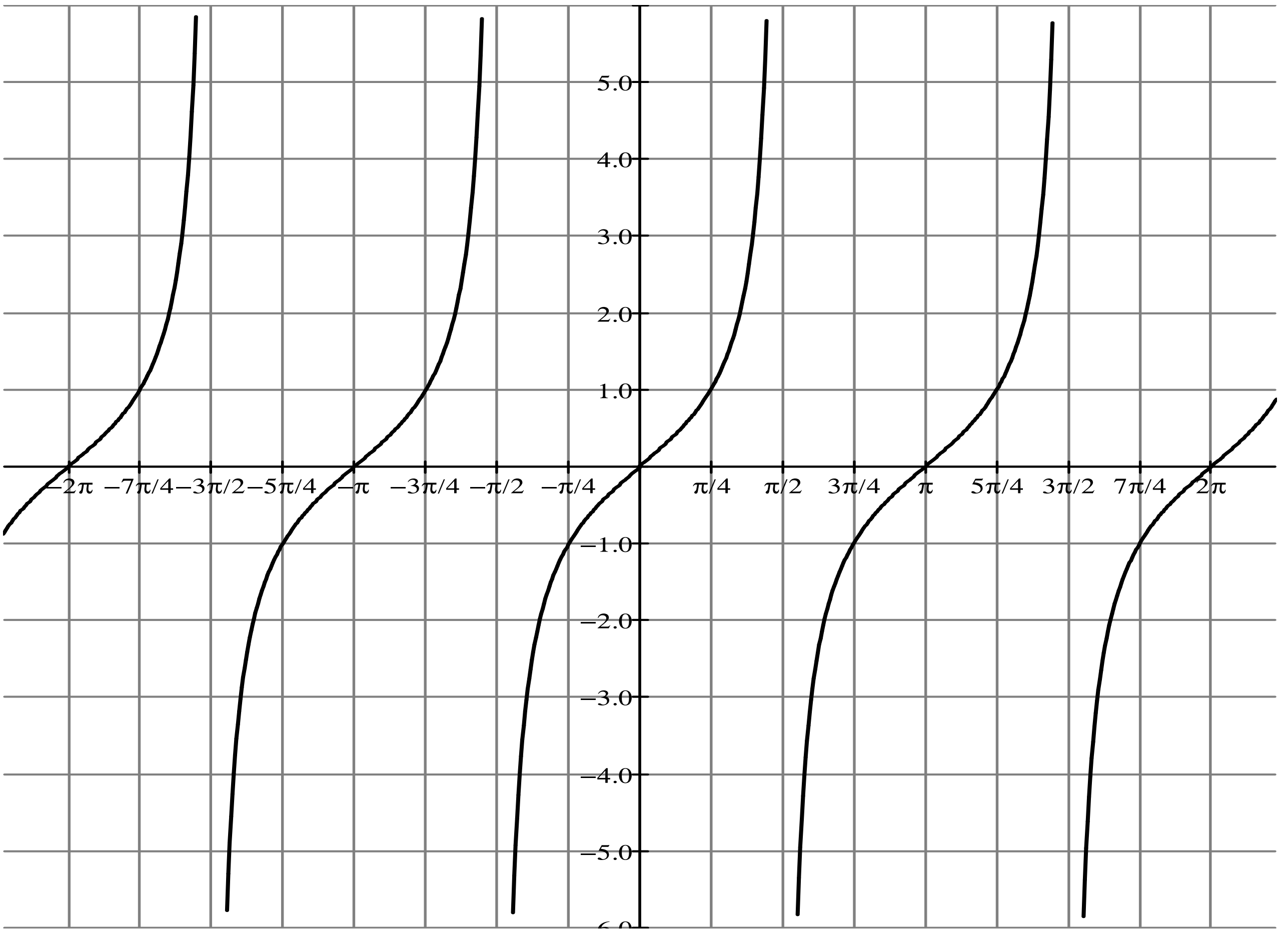
Below is the graph of $y = \cos x$. Recalling that $\sec x = 1/\cos x$, sketch the graph of $y = \sec x$ in the interval $x \in [-2\pi, 2\pi]$.

x	cos x	sec x
0	1	
π / 6	0.866	
π / 3	0.5	
π / 2	0	
2π / 3	-0.5	
5π / 6	-0.866	
π	-1	
7π / 6	-0.886	
4π / 3	-0.5	
3π / 2	0	
5π / 3	0.5	
11π / 6	0.866	
2π	1	



Below is the graph of $y = \tan x$. Recalling that $\cot x = 1/\tan x$, sketch the graph of $y = \cot x$ in the interval $x \in [-2\pi, 2\pi]$.

x	tan x	cot x
0	0	
π / 4	1	
π / 2	undef	
3π / 4	-1	
π	0	
5π / 4	1	
3π / 2	undef	
7π / 4	-1	
2π	0	

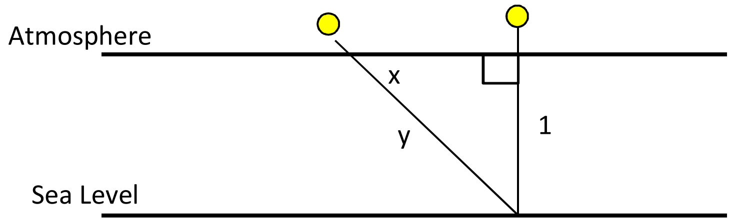


Complete the summary table.

Property	Cosecant $y = \csc x$	Secant $y = \sec x$	Cotangent $y = \cot x$
Domain			
Range			
Period			
Equations of Asymptotes			
Points of intersection with corresponding Primary Trig Functions			

Modelling with Reciprocal Relationships

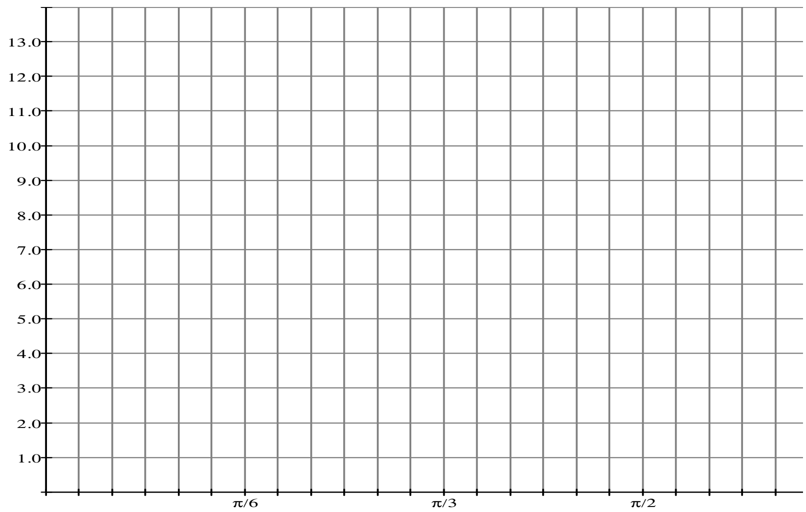
Example: When the sun is directly overhead, its rays pass through the atmosphere as shown. Call this 1 unit of atmosphere. When the Sun is not overhead, but is inclined at angle x to the surface of the Earth, its rays pass through more air before they reach sea level. Call this y units of atmosphere. The value of y affects the temperature of the Earth.



a) Determine an expression for y in terms of angle x.

b) Graph y = f(x) in the interval $x \in [0, \pi/2]$.

x	tan x	cot x
0	0	
π / 36	1	
π / 18	undef	
π / 12	-1	
π / 6	0	
π / 4	1	
π / 3	undef	
5π / 12	-1	
π / 2	0	



c) Describe what happens to the value of y as x approaches 0. Explain this answer in relation to the question.

Section 5.3 **Transformations of Sinusoidal Functions**

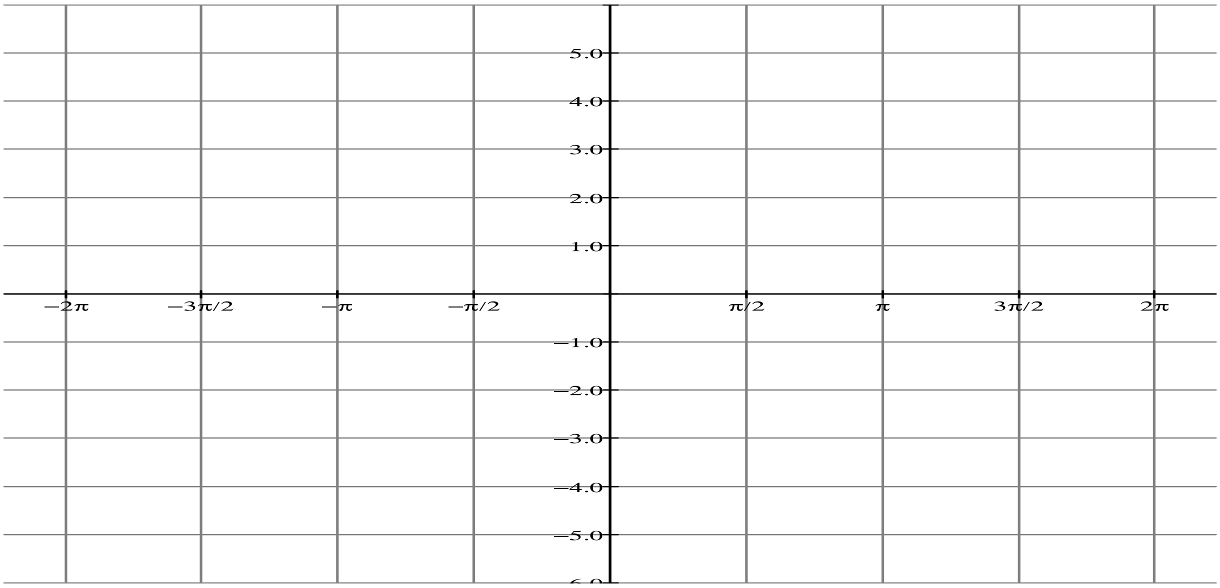
The transformation of a sine or cosine function f(x) to g(x) has the general form:

$g(x) = a\, f\, [k\, (x - d)] + c$

where |**a**| is the amplitude, if a < 0, there is a reflection in the x-axis
k is the horizontal stretch or compression, if k < 0 there is a reflection in the y-axis
d is the phase shift
c is the vertical displacement.

The period is given by 2π / k

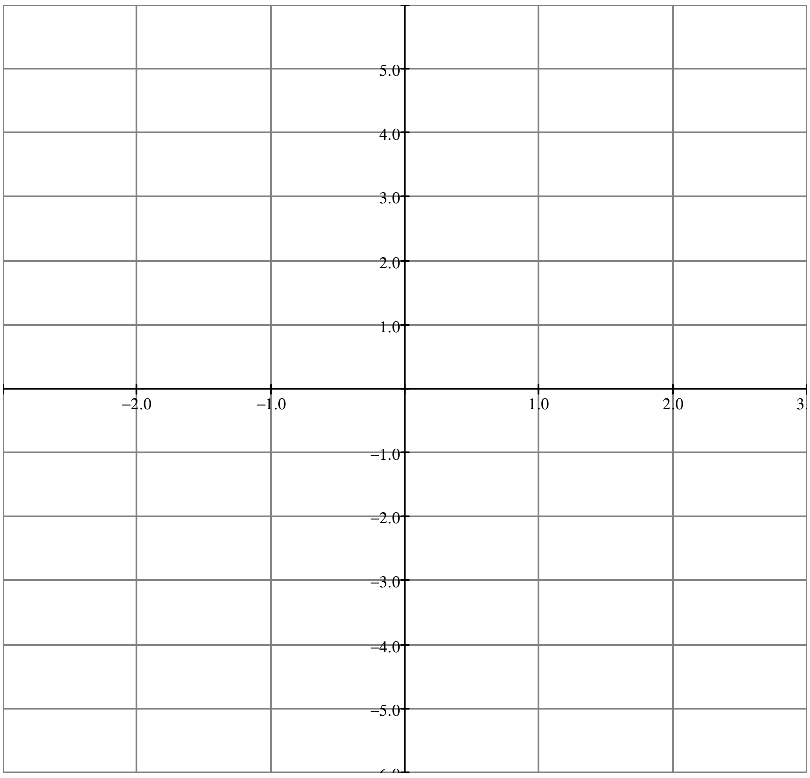
Examples: Transform the function $f(x) = \sin x$ to $g(x)$ such that $g(x) = 3\sin\left[\frac{3}{2}x + \frac{\pi}{4}\right] - 2$
State the min and max values, the amplitude, the period, the phase shift the vertical displacement, and the domain and range. Graph both functions, $f(x)$ and $g(x)$, over 2 cycles.



Transforming Sinusoidal functions to Match Data not Given in Terms of π

*Remember that a period, p, is p = 2π / k and that k = 2π / p.

Transform the function $f(x) = \cos x$ to $g(x)$ such that $g(x)$ has an amplitude of 2, a period of 1, a phase shift of 0.5 to the left and a vertical displacement of 3 units up. Graph the function over 2 cycles. Write the equation of the function.

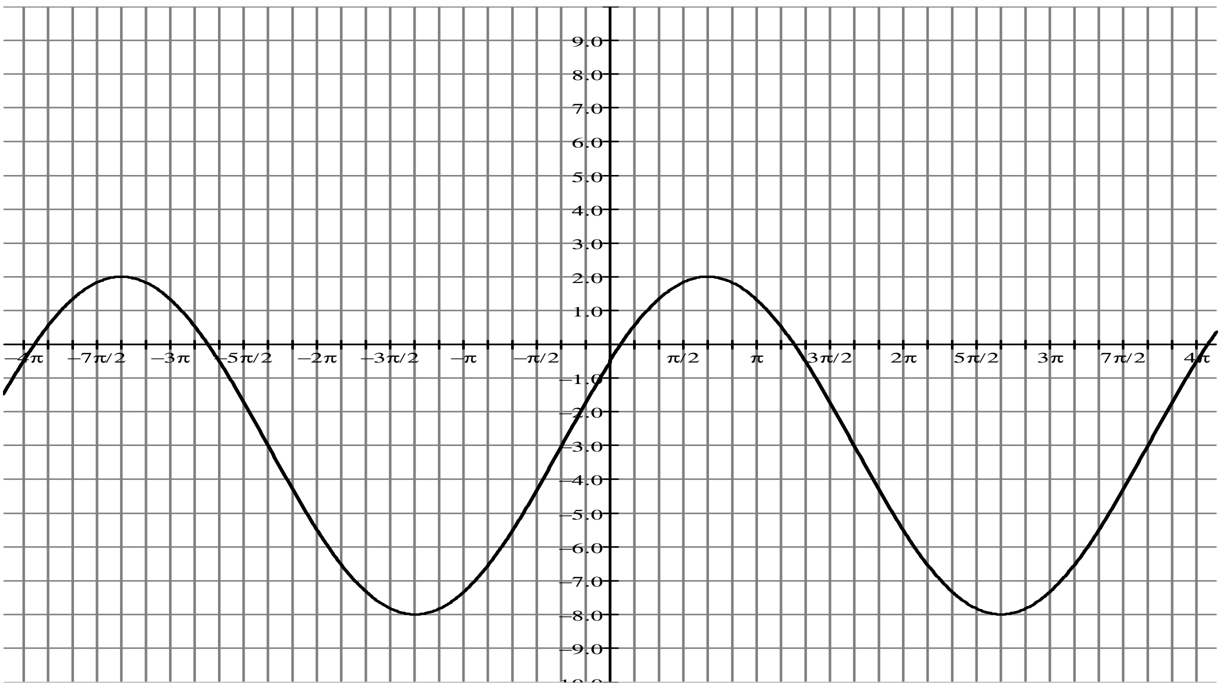


Example: Write an equation to represent the following functions.

a) A sine function with a maximum value of 5, a minimum value of -3, a phase shift of 5π/6 rad to the right and a period of 2π/3.

b) A cosine function has a maximum value of -2 and a minimum value of -3, a phase shift of 3 rad to the left and a period of 5.

Example: Given the graph below, write an equation using both a cosine and sine function.



Example: The vertical position, h, in metres, of a rider on a Ferris wheel, after time, t, in seconds, is a sinusoidal function. The maximum height above the ground is 22m and the minimum height is 2m. The Ferris wheel completes one turn in 30 seconds, and the model predicts the highest point at t = 0 seconds. Determine an equation to model the Ferris wheel's rotation as both a cosine and a sine function.

Section 5.4 **Solving Trigonometric Equations**

An equation that involves one or more trigonometric ratios of a variable is a trigonometric equation.

ex. $\sin\theta = 0.5$ ex. $4\cos x + 1 = 0$ ex. $2\tan 2x - 5\tan x - 3 = 0$

To solve linear trigonometric equations:

1) Isolate for sinθ, cosθ, tanθ, cscθ, secθ, or cotθ.
2) Switch any reciprocal trig ratios to their corresponding primary trig ratio.
3) Use the inverse function on your calculator or special triangles and the CAST rule to find θ.

Examples: Solve the following equations in the interval $x \in [-2\pi, 2\pi]$

a) Find the exact values of x, for $\sin x = -1/\sqrt{2}$

b) Round answers to 3 decimal places, for $\tan x - 3 = 0$

c) Round answers to 3 decimal places, for $2\sec x + 5 = 0$

To solve quadratic trigonometric equations:

1) Set one side equal to zero.

2) Let **a** = $\sin x$ or $\cos x$ or $\tan x$ or $\csc x$ or $\sec x$ or $\cot x$. Then replace the trig functions with **a** in the equation.

3) Factor the equation if possible. Then set each factor equal to zero and solve for **a**.

4) If it is not possible to factor, use the quadratic formula to solve for **a**.

5) Replace each **a** with the appropriate trig function.

6) Solve each factor using your rules for solving linear trigonometric equations.

Examples: Solve each equation in the interval $x \in [-2\pi, 2\pi]$

a) $\cos 2x - 1 = 0$ b) $2\csc 2x - \csc x - 1 = 0$ c) $5\cot 2x - 2\cot x - 3 = 0$

Example: The range of an arrow shot from a particular box can be modeled by the equation $r = 100\sin 2\theta$, where r is the range in metres and θ is the angle in radians above the horizontal that the arrow is released. A target is placed 80 m away.

a) What are the restrictions on the angle θ?

b) Determine the angle or angles that the archer should use to hit the target, to the nearest hundredth of a radian.

Section 5.5 **Making Connections and Instantaneous Rates of Change**

Instantaneous rates of change of a sinusoidal function follow a sinusoidal pattern.

Many real-world processes can be modelled with a sinusoidal function, even if they do not involve angles.

Modelling real-world processes usually require transformations of the basic sinusoidal functions.

Example 1: The height, h, in metres, of a car above the ground as a ferris wheel turns can be modelled using the function $h = 20sin(\pi t/60) + 25$, where t is the time, in seconds.

a) Determine the average rate of change of h over each time interval, rounded to 3 decimal places.

i) 5s to 10s ii) 9s to 10s iii) 9.9s to 10s iv) 9.99s to 10s

b) Estimate a value for the instantaneous rate of change of h at t = 10s.

c) What physical quantity does this instantaneous rate of change represent?

d) Would you expect the instantaneous rate of change of h to be the same at t = 15s? Justify you answer.

Example: The variations in maximum daily temperatures for Moose Factory, Ontario, on the first of the month from January to December are shown.

Month	Variation in Temp °C
1	-14
2	-14
3	-4.9
4	3.2
5	11
6	19.1
7	22.4
8	20.6
9	15.9
10	8.3
11	-1.7
12	-10

a) Write a sinusoidal function, (both a sine and a cosine function), to model the data.

b) Make a scatter plot of the data. Then graph one of your models on the same set of axes and compare the graphs.