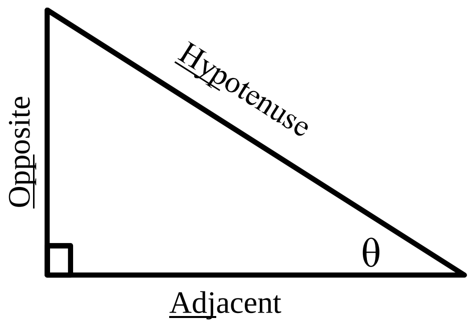
MHF4U Unit 4 Trigonometry

Section	Pages	Questions
Prereq Skills	200-201	#1ac, 2aceg, 3, 5bd, 6aceg, 7, 8, 9, 10, 11ac, 12, 13, 14
4.1	208-210	#1ac, 2ac, 3ac, 4ac, 5ace, 6ace, 7bcf, 8ace, 9, 10, 11, 13ab, 15, 17, 18
4.2	216-219	#1, 3bd, 4ac, 5bc, 6ac, 7, 8, 9, 10, 12, 13, 14, 16a(i), 17a(i), 18a
4.3	225- 227	#1, 2, 3, 4, 5, 6, 9, 10, 11, 14 * use the Unit Circle to derive the six co-function identities for x and (3pi/2-x), and for x and (3pi/2+x)
4.4	232-235	#1ac, 2bd, 3ac, 4a, 5a, 6a, 7a, 8, 9ad, 12, 13, 14, 15 ** Know your proofs of the compound angle formulas!!**
4.5	240-241	#1, 2, 3, 4, 5, 6, 7, 8, 9a, 10, 12, 13, 15, 16
Review (with calc)	244-245 246	# 1ac, 2ac, 3ac, 4ac, 5, 7, 15, 16 # 1, 2, 3, 8
Review (without calc)	244-245 246	# 8, 9, 10, 11, 12a, 13, 14bc, 17, 18, 19a, 20, 23 # 5, 6, 7, 9, 12, 20

Note: Questions with an asterisk* are extra questions that are optional for the daily homework. However, they are potential "extended-type" questions that could be included on a unit test.

Section 4.0 Prerequisite Skills

Primary Trig Ratios

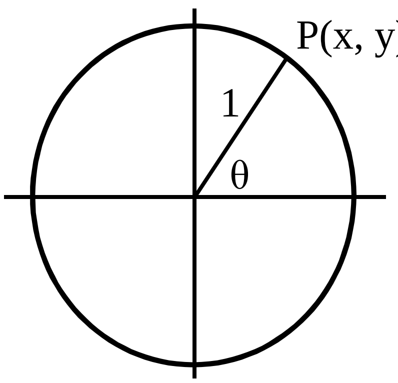


SOH $\sin\theta = \frac{opp}{hyp}$

CAH $\cos\theta = \frac{adj}{hyp}$

TOA $\tan\theta = \frac{opp}{adj}$

Unit Circle



P(x, y) =P(cosθ, sinθ)

$\tan\theta = \frac{sin\theta}{cos\theta}$

CAST Rule

S Sine Ratio is + \`ve	A All Trig Ratios are + \`ve
T Tan Ratio is + \`ve	C Cos Ratio is + \`ve

The CAST rule reminds us that for any trig ratio there are two possible angle positions between 0° and 360° where the ratio can give the same result.

To solve for both angles between 0° and 360°:

- $Sin\theta \ = \ k$
  $\theta_1 = sin^{-1}(k)$
  $\theta_2 = 180° - \ \theta_1$

- $Cos\theta \ = \ k$
  $\theta_1 = cos^{-1}(k)$
  $\theta_2 = 360° - \ \theta_1$

- $Tan\theta \ = \ k$
  $\theta_1 = tan^{-1}(k)$
  $\theta_2 = 180° + \ \theta_1$

Examples: An exact value for a trigonometric ratio is given for each angle. Determine the exact values of the other two primary trigonometric ratios.

a) $sin\theta = \frac{5}{13}, 0° \leq \theta \leq 90°$

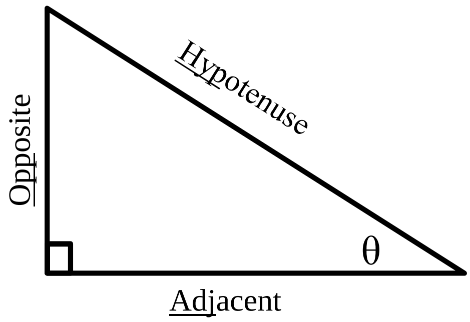
b) $cos\theta = \frac{5}{6}, 270° \leq \theta \leq 360°$

c) $tan\theta = -\frac{15}{17}, 90° \leq \theta \leq 180°$

Examples: Solve the following for both angles between 0° and 360°. Round to the nearest degree.

a) $sin\theta = 0.423$ b) $cos\theta = -0.676$ c) $tan\theta = 4.259$

Reciprocal Trig Ratios (Secondary Trig Ratios**)**



$$\csc\theta = \frac{hyp}{opp},\ \sec\theta = \frac{hyp}{adj},\ \cot\theta = \frac{adj}{opp}$$

$$\csc\theta = \frac{1}{sin\theta},\ \sec\theta = \frac{1}{cos\theta},\ \cot\theta = \frac{1}{tan\theta}$$

** To use our calculators with the reciprocal trig ratios, we must use the reciprocal key, 1/x or x^{-1} along with sine, cosine, or tangent keys.**

Examples: An exact value for a reciprocal trigonometric ratio is given for each angle. Determine the exact values of the other two reciprocal trigonometric ratios.

a) $csc\theta = \frac{12}{7}, 0° \leq \theta \leq 90°$

b) $sec\theta = \frac{17}{15}, 270° \leq \theta \leq 360°$

c) $cot\theta = -\frac{5}{13}, 90° \leq \theta \leq 180°$

Example: Determine the following trig ratios to 3 decimal places.

a) $\csc 36° =$ b) $\sec 80° =$ c) $\cot 52° =$

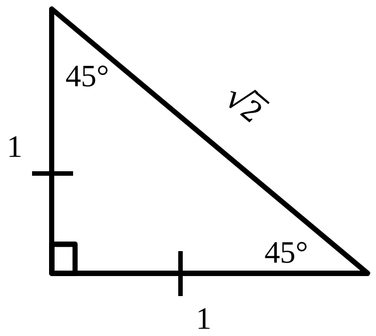
Examples: Solve the following for both angles between 0° and 360°. Round to the nearest degree.

a) $csc\theta = 2.564$ b) $sec\theta = 3.723$ c) $cot\theta = -1.149$

Exact Trigonometric Ratios of Special Angles

A reference angle is the acute angle between the terminal arm and the x-axis.

Angle 45° and its multiples (135°, 225°, 315°)



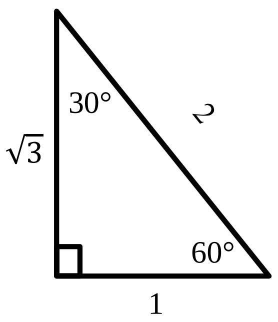
$\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

Use the CAST rule to determine the sign of the trigonometric ratio.

Example: Determine the EXACT VALUES of the sin, cos, and tan of the following angles.

a) θ = 135° b) θ = - 45° c) θ = 225°

Angle 30° (150°, 210°, 330°) and 60° (120°, 240°, 300°) and their multiples:



$\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

Example: Determine the EXACT VALUES of the sin, cos, and tan of the following angles.

a) θ = 150° b) θ = 240° c) θ = -150° d) θ = -300°

Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Example: Determine the distance between the points (-2, 14) and (5, -9).

Working With Radicals

Radical Sign: the symbol $\sqrt{}$ denotes the positive square root of a number.

Radicand: a number or expression under the radical sign.

Entire Radical: a radical in the form, $\sqrt{n}$, where n > 0. Ex. $\sqrt{24}$

Mixed Radical: a radical in the form $a\sqrt{b}$, where a ≠ 1 or -1, and b > 0. Ex. $2\sqrt{6}$

Multiplication Property of Radicals: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, where a ≥ 0, b ≥ 0

* **Change Entire Radical to Mixed Radicals/ Write Radicals in Simplest Form**:

1) Look for the largest perfect square factor
2) Use the multiplication property to write the radical in simplest form (i.e. Write as a "mixed radical").

Example: Express the following as a mixed radical in simplest form.

a) $\sqrt{32}$ b) $\sqrt{40}$ c) $\sqrt{50}$ d) $\sqrt{300}$

Adding and Subtracting Radicals

To add and subtract radicals, you must have **like radicals**.

Like radicals have the **same radicand**, i.e. same number under the radical sign

For example, $\sqrt{6}$ and $3\sqrt{6}$ are like radicals, whereas $\sqrt{10}$ and $\sqrt{7}$ are NOT like radicals

Sometimes we can make like radicals by rewriting radicals in simplest form.

$$\left.\begin{array}{l} \text{Ex. } 5\sqrt{8} \text{ and } 2\sqrt{18} \rightarrow 5\sqrt{8} = 5\sqrt{4x2} = 5\sqrt{4}\sqrt{2} = 10\sqrt{2} \\ \rightarrow 2\sqrt{18} = 2\sqrt{9x2} = 2\sqrt{9}\sqrt{2} = 6\sqrt{2} \end{array}\right\} \text{Now like radicals}$$

When adding and subtracting like radicals:

1) Add/subtract the coefficients in front of the radical. *The number under the radical sign stays the same. (The radicand remains constant)

2) Ensure all radicals in your final answer are written in simplest form.

Examples: Simplify and collect like radicals.

a) $\sqrt{18}+\sqrt{75}-\sqrt{27}+\sqrt{8}$

b) $6\sqrt{20}+4\sqrt{54}-5\sqrt{24}-2\sqrt{125}$

c) $\frac{1}{4}\sqrt{12}-3\sqrt{28}+\frac{5}{8}\sqrt{48}+\frac{2}{3}\sqrt{63}$

Multiplication and Division Properties of Radicals:

$\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

where a ≥ 0, b ≥ 0

$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

where a ≥ 0, b ≥ 0

Example: Simplify the following radicals

a) $\sqrt{11} \times \sqrt{6}$

b) $\frac{\sqrt{35}}{\sqrt{5}}$

c) $2\sqrt{13} \times 3\sqrt{2}$

d) $\frac{4\sqrt{10}}{2\sqrt{2}}$

e) $\frac{8-\sqrt{20}}{2}$

f) $3\sqrt{2}(2\sqrt{3} - 5) - \sqrt{6}(1 - 4\sqrt{3})$

g) $(2 + 4\sqrt{3})(2 - 4\sqrt{3})$

h) $(\sqrt{3} + 6)(5 - \sqrt{3})$

Example: Determine the exact perimeter and exact area of a rectangle with length 7 cm and diagonal 9 cm.

Rationalizing the Denominator

Fractions should not be left with radicals in the denominator (just like how we should not have decimals in fractions). Rationalizing the denominator is the process in which we remove the radical from the denominator (similar to finding the common denominator when working with fractions).

If there is only the radical in the denominator:

1. Multiply the numerator and denominator by the radical. $\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$

2. Simplify the numerator and the denominator. $\frac{a\sqrt{b}}{\sqrt{b^2}} = \frac{a\sqrt{b}}{b}$

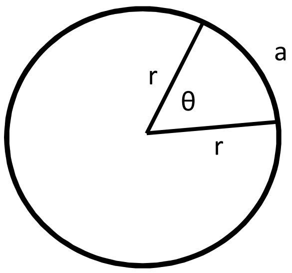
Example: Rationalize the following denominators

a) $\frac{2}{\sqrt{3}}$ b) $\frac{3\sqrt{2}}{\sqrt{3}}$ c) $\frac{2+\sqrt{3}}{\sqrt{2}}$

Section 4.1 Radian Measure

The radian measure of an angle θ is defined as the length, a, of the arc that subtends the angle divided by the radius, r, of the circle.

$$\theta = \frac{a}{r}$$



For one complete revolution, the length of the arc equals the circumference of the circle, 2πr.

$$\theta = \frac{2\pi r}{r} = 2\pi$$

One complete revolution measures 2π radians.

Converting Radians to Degrees

$$2\pi\ radians = 360°$$

$$1\ radian = \frac{360°}{2\pi}$$

$$1\ radian = \frac{180°}{\pi}$$

Multiply the radians by $\frac{180°}{\pi}$ to determine the equivalent degree measurement.

∴ 1 radian is approx. 57.3^{o}.

Converting Degrees to Radians

$$360° = 2\pi\ radians$$

$$1° = \frac{2\pi}{360°}$$

$$1° = \frac{\pi}{180°}$$

Multiply the degrees by $\frac{\pi}{180°}$ to determine the equivalent radian measurement.

∴ 1^{o} is approx. 0.0175 radians.

Examples:

a) Convert 60^o to radians. Determine an exact measurement and an approximate measurement.

b) Using the exact value for 60^o, determine the radian measure of 120^o & 20^o.

c) Convert $\frac{2\pi}{3}$ radians to degrees.

d) Convert 4.32 radians to degrees.

Arc Length

To determine the arc length, we use the formula $\theta = \frac{a}{r}$ and isolate for **a,** $a = r\theta$.
To use the arc length formula θ must be measured in radians.

Example: A circle has a radius of 4.7cm. Determine the length of the arc subtended by each angle.

a) 1.7 radians

b) 64^o

Angular Velocity of a Rotating Object

The angular velocity of a rotating object is the rate at which the central angle changes with respect to time.

To determine the Angular Velocity of a Rotating Object:

1) Multiply the number of revolutions by 360o or 2π depending on whether the answer is required in degrees or radians.

2) Divide your answer by the corresponding unit of measurement.

Example: The hard disk in a computer rotates at 7200 revolutions per minute. Determine the angular velocity in:

a) Degrees per second.

b) Radians per second.

Section 4.2 Trigonometric Ratios and Special Angles

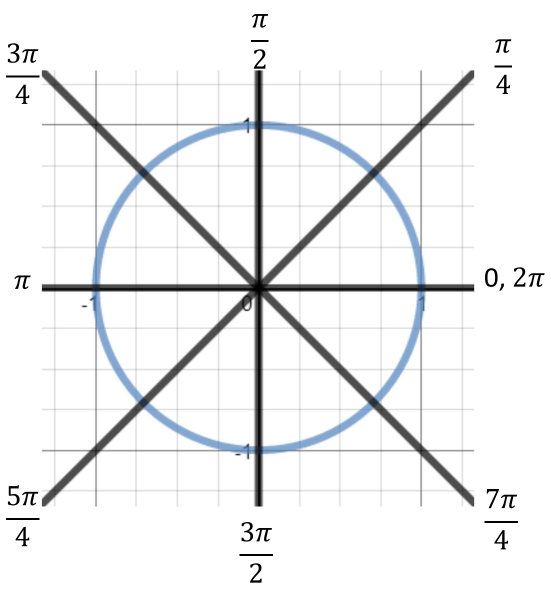
To determine trigonometric ratios of angles measured in radians, you must switch your calculator to RADIAN (RAD,R) Mode.

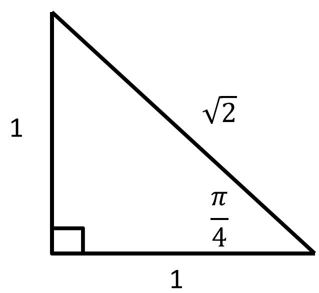
Examples: Use a calculator to determine the following trigonometric ratios to 3 decimal places.

a) $\sin\frac{\pi}{6}$ b) $\cos\frac{\pi}{4}$ c) $\tan\frac{2\pi}{3}$

d) $\csc\frac{\pi}{6}$ e) $\sec\frac{\pi}{4}$ f) $\cot\frac{2\pi}{3}$

Special Angles between 0 and 2π are displayed on the unit circle below increments of $\pi/4$ (45^{o}). These special angles and the CAST rule are useful in solving trigonometry problems without using a calculator.





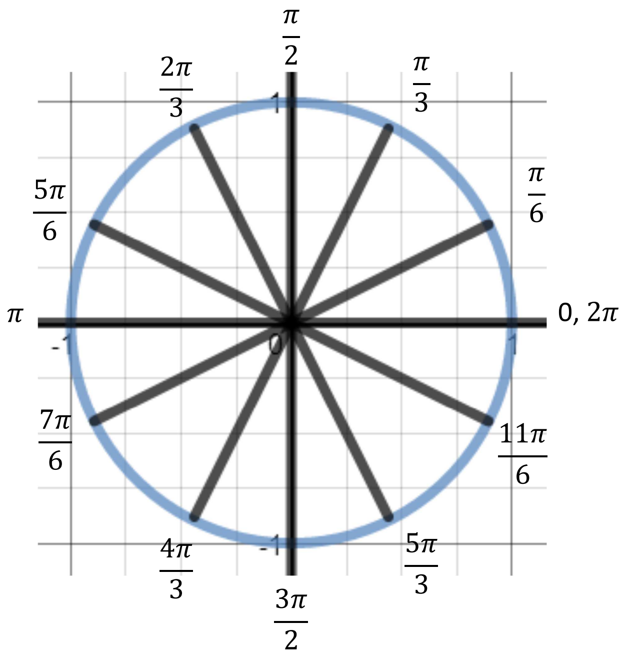
Example: Determine the following exact trigonometric ratios.

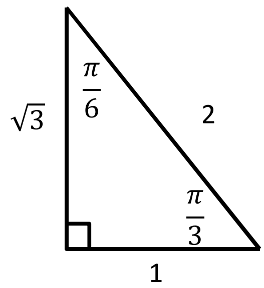
a) $\sin\frac{7\pi}{4} =$

b) $\sec\frac{\pi}{4} =$

c) $\tan\frac{3\pi}{4} =$

Special angles between 0 and 2π are displayed on the unit circle below in increments of π/6 (30°) and/or π/3 (60°). These special angles and the CAST rule are useful in solving trigonometry problems without using a calculator.





Example: Determine the following exact trigonometric ratios.

a) $\sin\frac{\pi}{6} =$ b) $\tan\frac{2\pi}{3} =$ c) $\cot\frac{5\pi}{3} =$

To solve problems using exact trigonometric ratios, you can use the properties of similar triangles.

Recall: Similar triangles have the same angles and corresponding sides are proportional.

Example: Sarah is flying a kite at the end of a 30m string. The sun is directly overhead and the string makes an angle of $\pi/6$ with the ground. Suddenly the wind speed increases and the kite flies higher until the string makes an angle of $\pi/3$ with the ground.

a) Determine an exact expression for the horizontal distance of the shadow when the kite is in position 1.

b) Determine an exact expression for the horizontal distance of the shadow when the kite is in position 2.

c) Determine an exact expression for the horizontal distance that the shadows moves between the 2 positions of the kite.

d) Determine the approximate distance from part c) to the nearest tenth of a meter.

Section 4.3 Equivalent Trigonometric Expressions

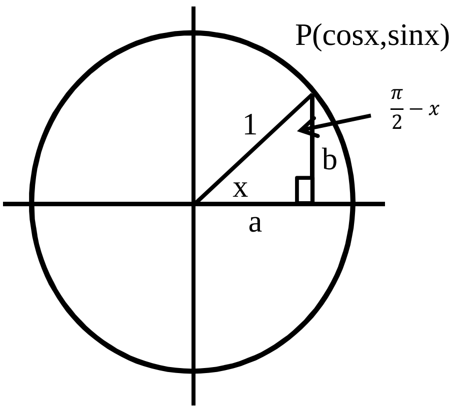
- Equivalent trigonometric expressions are expressions that yield the same value for all values of the variable.

- An identity is an equation that is true for all values of the variable for which the expressions on both sides of the equation are defined.

- An identity involving trigonometric expressions is called a trigonometric identity.

Cofunction Identities

Using a Right Triangle and the Unit Circle to determine equivalent trigonometric identities featuring $\frac{\pi}{2}$.



$\sin x = b$	$\sin\left(\frac{\pi}{2}-x\right) = a$
$\cos x = a$	$\cos\left(\frac{\pi}{2}-x\right) = b$
$\tan x = \frac{b}{a}$	$\tan\left(\frac{\pi}{2}-x\right) = \frac{a}{b}$
$\csc x = \frac{1}{b}$	$\csc\left(\frac{\pi}{2}-x\right) = \frac{1}{a}$
$\sec x = \frac{1}{a}$	$\sec\left(\frac{\pi}{2}-x\right) = \frac{1}{b}$
$\cot x = \frac{a}{b}$	$\cot\left(\frac{\pi}{2}-x\right) = \frac{b}{a}$

So in Quadrant 1

$$\sin\left(\frac{\pi}{2}-x\right) = \cos x$$

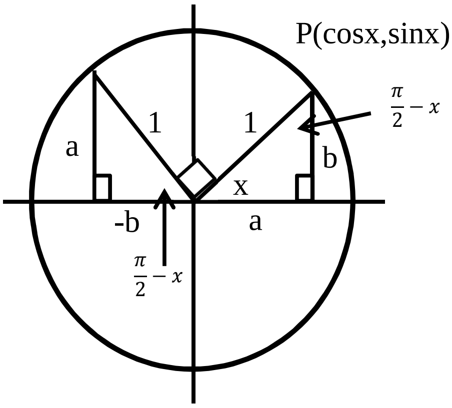
$$\cos\left(\frac{\pi}{2}-x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2}-x\right) = \cot x$$

$$\csc\left(\frac{\pi}{2}-x\right) = \sec x$$

$$\sec\left(\frac{\pi}{2}-x\right) = \csc x$$

$$\cot\left(\frac{\pi}{2}-x\right) = \tan x$$



$\sin x = b$

$\cos x = a$

$\tan x = \frac{b}{a}$

$\csc x = \frac{1}{b}$

$\sec x = \frac{1}{a}$

$\cot x = \frac{a}{b}$

$\sin\left(\frac{\pi}{2}+x\right) = a$

$\cos\left(\frac{\pi}{2}+x\right) = -b$

$\tan\left(\frac{\pi}{2}+x\right) = -\frac{a}{b}$

$\csc\left(\frac{\pi}{2}+x\right) = \frac{1}{a}$

$\sec\left(\frac{\pi}{2}+x\right) = -\frac{1}{b}$

$\cot\left(\frac{\pi}{2}+x\right) = -\frac{b}{a}$

So in Quadrant 2

$$\sin\left(\frac{\pi}{2}+x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$$

$$\tan\left(\frac{\pi}{2}+x\right) = -\cot x$$

$$\csc\left(\frac{\pi}{2}+x\right) = \sec x$$

$$\sec\left(\frac{\pi}{2}+x\right) = -\csc x$$

$$\cot\left(\frac{\pi}{2}+x\right) = -\tan x$$

Example: Given that $\sin\frac{\pi}{5} \cong 0.5878$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

a) $cos\,\frac{3\pi}{10}$

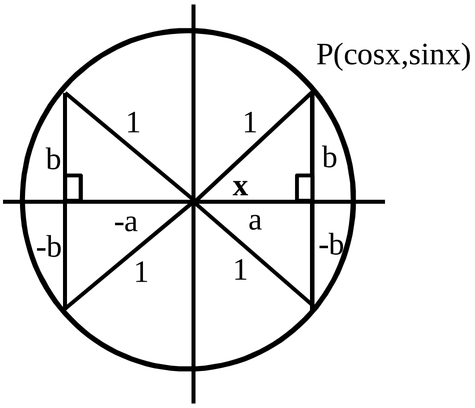
b) $cos\,\frac{7\pi}{10}$

Example: Given that $\csc\frac{2\pi}{7} \cong 1.2790$, use equivalent trigonometric expressions to determine the $\sec\frac{3\pi}{14}$, to four decimal places.

Example: Given that $\cot\frac{\pi}{6} = \sqrt{3}$, use equivalent trigonometric expressions to show that $\tan\frac{2\pi}{3} = -\sqrt{3}$, to four decimal places.

Example: Given that $\sec \mathrm{b} = \csc 1.05$, and that b lies in the first quadrant, use a cofunction identity to determine the measure of angle b, to two decimal places.

More Cofunction Identities



In Quadrant I:

$\sin x = b$
$\cos x = a$
$\tan x = b/a$
$\csc x = 1/b$
$\sec x = 1/a$
$\cot x = a/b$

In Quadrant II:

$sin(\pi - x) = b$
$cos(\pi - x) = -a$
$tan(\pi - x) = -b/a$
$csc(\pi - x) = 1/b$
$sec(\pi - x) = -1/a$
$cot(\pi - x) = -a/b$

In Quadrant III:

$sin(\pi + x) = -b$
$cos(\pi + x) = -a$
$tan(\pi + x) = b/a$
$csc(\pi + x) == 1/b$
$sec(\pi + x) = -1/a$
$cot(\pi + x) = a/b$

In Quadrant IV:

$sin(2\pi - x) = -b$
$cos(2\pi - x) = a$
$tan(2\pi - x) = -b/a$
$csc(2\pi - x) = -1/b$
$sec(2\pi - x) = 1/a$
$cot(2\pi - x) = -a/b$

Section 4.4 Compound Angles

A trigonometric expression that depends on two or more angles is known as a compound angle expression.

Trig ratio's do not have a distributive property –

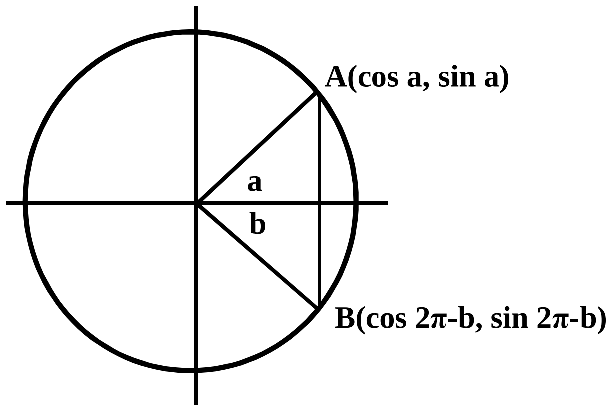
Ex. $2(x + 1) = 2x + 2$

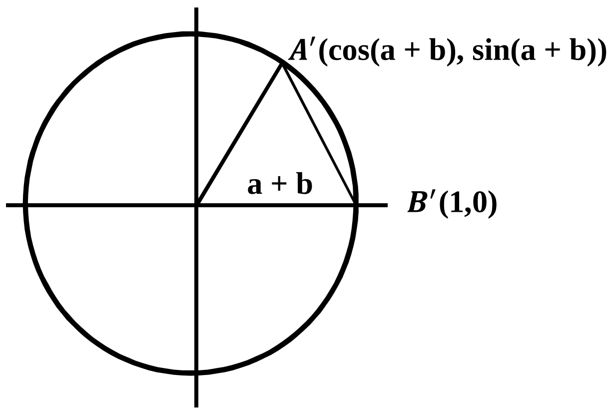
You can't just say that $\sin (x + 1) = \sin x + \sin 1$ – pick any angle combination and test for yourself, you will see that it simply does not work.

For trig ratio's expansion of double angles is a little more complex.

Let's start with the **Addition Formula for Cosine:**

Picture a unit circle with two angles, **a** and **b** which connect the center of the circle to points **A**, **B** respectively as shown below. We could connect these two points with secant $\overline{AB}$.





Let's rotate the whole thing by angle **b,** that way we have a nicer starting point, an ending point that has coordinates with the relation we want to find [cos(a + b)] and a secant distance $\overline{A'B'}$ that has to be exactly the same distance as secant $\overline{AB}$.

Pythagorean Theorem Time – let us calculate the length of both secants and set them equal to each other.

$\overline{AB} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ **A(cos a, sin a) , B(cos 2π-b, sin 2π-b)**

Before we start we are going to make use of a couple of the identities we created in the last section. Specifically $cos(2\pi - x) = \cos x$ and $sin(2\pi - x) = -\sin x$ to be used in point B, you can thank me later ☺.

$$\overline{AB} = \sqrt{(-\sin b - \sin a)^2 + (\cos b - \cos a)^2}$$

$\overline{A'B'} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ **A'(cos(a + b), sin(a + b)) , B'(1,0)**

$$\overline{A'B'} = \sqrt{(0 - \sin(a + b))^2 + (1 - \cos(a + b))^2}$$

Now let us set the equal and expand.

$$\sqrt{(-\sin b - \sin a)^2 + (\cos b - \cos a)^2} = \sqrt{(1 - \sin(a + b))^2 + (0 - \cos(a + b))^2}$$

$$(-\sin b - \sin a)^2 + (\cos b - \cos a)^2 = (1 - \sin(a + b))^2 + (0 - \cos(a + b))^2$$

$$\sin^2 b + 2\sin a \sin b + \sin^2 a + \cos^2 b - 2\cos a \cos b + \cos^2 a$$
$$= \sin^2(a + b) + 1 - 2\cos(a + b) + \cos^2(a + b)$$

Recall $\sin^2(x) + \cos^2(x) = 1$ Pythagorean Identity

So

$\underline{\sin^2 b} + 2\sin a \sin b + \underline{\sin^2 a} + \underline{\cos^2 b} - 2\cos a \cos b + \underline{\cos^2 a} = \underline{\sin^2(a+b)} + 1 - 2\cos(a+b) + \underline{\cos^2(a+b)}$

Becomes

$1 + 1 + 2\sin a \sin b - 2\cos a \cos b = 1 + 1 - 2\cos(a+b)$

$2\sin a \sin b - 2\cos a \cos b = -2\cos(a+b)$

$-\sin a \sin b + \cos a \cos b = \cos(a+b)$

Or $\boxed{\boldsymbol{\cos(a+b) = \cos a \cos b - \sin a \sin b}}$

Subtraction Formula for Cosine **(**we will take the lazy approach**)**

If $\boldsymbol{\cos(a+b) = \cos a \cos b - \sin a \sin b}$, and we want cos (a – b), let's just sub in a negative b value into the formula.

$\boldsymbol{\cos(a+(-b)) = \cos a \cos(-b) - \sin a \sin(-b)}$

$\boldsymbol{\cos(a-b) = \cos a \cos(2\pi - b) - \sin a \sin(2\pi - b)}$

$\boldsymbol{\cos(a-b) = \cos a \cos(2\pi - b) - \sin a \sin(2\pi - b)}$

$\boldsymbol{\cos(a-b) = \cos a \cos b - \sin a\,(-\sin\ b)}$

$\boxed{\boldsymbol{\cos(a-b) = \cos a \cos b + \sin a \sin b}}$

Addition Formula for Sine

Recall

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cos(-x) = \cos x$$

$$\sin(-x) = -\sin x$$

So

$$\sin\,(\mathrm{a} + \mathrm{b}) = \cos\left(\frac{\pi}{2} - (a + b)\right)$$

$$\sin\,(\mathrm{a} + \mathrm{b}) = \cos\left((\frac{\pi}{2} - a) - b\right)$$

$$\boldsymbol{sin(a + b) = \cos(}\frac{\pi}{2} - a)\ \boldsymbol{\cos b + \sin(}\frac{\pi}{2} - a)\ \boldsymbol{\sin b}$$

$$\boxed{\boldsymbol{sin(a + b) = \sin a \cos b + \cos a \sin b}}$$

Subtraction Formula for Sine

Let b = - b as we did for cosine - yours to finish.

Example: Show that the formula $cos(x - y) = cosxcosy + sinxsiny$ is true for $x = \pi/6$ and $y = \pi/3$

Example: Show that the formula $sin(x + y) = sinxcosy + cosxsiny$ is true for $x = \pi/2$ and $y = 3\pi/4$

Examples: Use an appropriate compound angle formula to express the following as a single trigonometric function and then determine an exact value for each.

a) $\sin\frac{\pi}{4}\cos\frac{\pi}{12} - \cos\frac{\pi}{4}\sin\frac{\pi}{12}$

b) $\cos\frac{10\pi}{9}\cos\frac{5\pi}{18} + \sin\frac{10\pi}{9}\sin\frac{5\pi}{18}$

Example: Use an appropriate compound formula to determine an exact value for $\cos\frac{5\pi}{12}$

Example: Angles x and y are located in the first quadrant such that sin x $=\frac{5}{13}$ and cos $y = \frac{3}{5}$
Determine an exact value for $sin(x - y)$.

Example: Use an appropriate compound angle formula to

a) Prove the double angle formula for sine, $sin2\theta = 2sin\theta cos\theta$.

b) Prove the double angle formula for cosine, $cos2\theta = cos^2\theta - sin^2\theta$
or $= 2cos^2\theta - 1$
or $= 1 - 2sin^2\theta$

Section 4.5 Proving Trigonometric Identities

In this section, you will use the following basic trigonometric identities to prove other identities.

The Pythagorean Trig Identity (PTI):

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$

Reciprocal Trig Identities (RTI):

$$\csc\theta = \frac{1}{sin\theta},\ \sec\theta = \frac{1}{cos\theta},\ \cot\theta = \frac{1}{tan\theta}$$

Quotient Trig Identities (QTI):

$$\tan\theta = \frac{\sin\theta}{cos\theta},\ \cot\theta = \frac{\cos\theta}{sin\theta}$$

Compound Angle Formulas:

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

Double Angle Formulas:

$$sin2\theta = 2sin\theta cos\theta$$
$$cos2\theta = cos^2\theta - sin^2\theta$$
$$cos2\theta = 2cos^2\theta - 1$$
$$cos2\theta = 1 - 2sin^2\theta$$

Example: Prove $cos\left(\frac{\pi}{2} - x\right) = \sin x$

The goal is to show that the Left Hand Side (LHS) = Right Hand Side (RHS)

LHS:	RHS:
$cos\left(\frac{\pi}{2} - x\right)$	$\sin x$
$\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$	$\sin x$
$0(\cos x) + 1(\sin x)$	$\sin x$
$\sin x$	$\sin x$

Therefore LHS = RHS

Example: Prove $\csc 2\theta = \frac{\csc\theta}{2\cos\theta}$

RHS: $\frac{\csc\theta}{2\cos\theta} = \csc\theta\left(\frac{1}{2\cos\theta}\right)$

$$= \frac{1}{\sin\theta}\left(\frac{1}{2\cos\theta}\right)$$

$$= \frac{1}{2\sin\theta\cos\theta}$$

$$= \frac{1}{\sin 2\theta}$$

$$= \csc 2\theta$$

$$= LHS$$

Therefore LHS = RHS

Example: Prove that $\cos(x+y)\cos(x-y) = \cos^2 x + \cos^2 y - 1$

Example: Prove that $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} - \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = 2\tan 2\theta$