MHF4U Unit 1 Polynomial Functions

Section	Pages	Questions
Prereq Skills	2 – 3	# 1ace, 2cde, 3bce, 4, 5, 6, 7, 8ace, 9, 10b, 11b, 12 & Factoring Practice
1.1	11 – 14	# 1, 2, 3, 4, 5, 7, 8, 9(in class)
1.2	26 – 29	# 1, 2, 3, 4abcf, 5, 6, 7, 8, 11(in class), 12
1.3	39 – 41	# 1bc, 2ab, 3, 5, 6ac, 7bd, 9(don't graph), 11a, 12ab, 14*
1.4	49 – 52	# 1acd, 2, 3, 4, 5, 6, 7abc, 8ac, 9, 10, 12, 14*
1.5	62 – 64	# 1, 2, 3, 4, 5, 7a(don't graph)bcd, 10ab
1.6	71 – 73	# 1, 2, 3, 4, 5, 7, 9, 10*, 11*
Review	74 – 77 79	# 1-11, 12(don't graph), 13, 14, 15, 17, 18 # 8abd, 13

Note: Questions with an asterisk* are extra questions that are optional for the daily homework. However, they are potential "extended-type" questions that could be included on a unit test.

Unit 1 - Lesson 1

Prerequisite Skills

Grade 9

- Slope
- equation of a straight line
- x intercept
- y intercept
- first differences

Grade 10

- factoring
- quadratic equation in vertex form
- second differences
- basic transformations of quadratic
- distance between two points in space

Grade 11

- function notation
- transformations of functions
- domain and range
- vertical asymptotes and holes
- radicals and rational functions
- sketching functions

Function Notation

To represent functions, we use notations such as $f(x)$ and $g(x)$.

ex. Linear function: $y = 2x + 1$

In Function notation: $f(x) = 2x + 1$

The notation $f(x)$ is read "f of x" or "f at x".
It means that the expression that follows contains x as a variable.

For example: $f(3)$ means substitute 3 for every x in the expression and solve for y, or $f(3)$.

ex. Quadratic function:

$$y = x^2 - 4x + 7$$
$$f(3) = (3)^2 - 4(3) + 7$$
$$f(3) = 9 - 12 + 7$$
$$f(3) = 4$$

Therefore when $x = 3$, $y = 4$ or $f(3) = 4$

For example: $f(2a)$ means substitute 2a for every x in the expression and solve for y, or $f(2a)$.

ex. Linear function: $y = 3x - 10$ find $f(2a)$

$$f(2a) = 3(2a) - 10$$
$$f(2a) = 6a - 10$$

Therefore when $x = 2a$, $y = 6a - 10$ or $f(2a) = 6a - 10$
we can create new equations or functions

Examples: Determine each value for the function $f(x) = x^2 - 4x + 1$

a) $f(0)$ b) $f(-2)$ c) $f(1/2)$

d) $f(3x)$ e) $-2f(2x)$

Slope and y-intercept of a line

The equation of a line, written in the form y = mx + b has m = slope and b = y-intercept

Examples: Determine the slope and y-intercept of the following lines.

a) y = 3x -1

m = 3

b = - 1

b) 2x - 7y = 14

7y = 2x – 14

y = 2/7x – 2

m = 2/7

b = - 2

c) y + 2 = 7(x - 1)

y + 2 = 7x - 7

y = 7x – 9

m = 7

b = - 9

Equation of a Line (y = mx + b)

To write the equation of a line, you need the slope and the y-intercept
Recall: The Slope Formula
Given two points (x_1, y_1) and (x_2, y_2), the slope is given by: $m = \frac{\Delta y}{\Delta x}, m = \frac{y_2 - y_1}{x_2 - x_1}$

Examples: Determine the equation of the line that satisfies each set of conditions.

a) Slope is -1 and the y-intercept is 7.

y = -1x + 7

b) Slope is 2 and it passes through the point (1, -4).

y = 2x + b

-4 = 2(1) + b

-4 = 2 + b

-6 = b

y = 2x – 6

or

y = m(x – p) + q

y = 2(x – 1) - 4

y = 2x – 2 – 4

y = 2x – 6

c) Line passes through the points (-2, 0) and (2, 4).

$m = \frac{4-0}{2-(-2)} = \frac{4}{4} = 1$

y = m(x – p) + q

y = 1(x – 2) + 4

y = 1x – 2 + 4

y = 1x + 2

Finite Differences

Finite differences can be used to determine whether a function is linear, quadratic or neither.
Finite differences can ONLY be used if the x-values in the table are increasing/decreasing by the same amount.
If the 1st differences are constant, the function is linear.
If the 2nd differences are constant, the function is quadratic.

Ex: Use finite differences to determine whether the functions below are linear, quadratic, or neither.

a)

X	Y	1st Diff	2nd Diff
1	4		
2	1		
3	0		
4	1		
5	4		

b)

X	Y	1st Diff	2nd Diff
-2	9		
-1	7		
0	5		
1	3		
2	1		

Interval Notation:

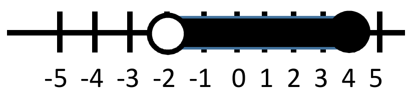
- Used to express a set of numbers
- Intervals that are infinite are ∞ and -∞
- Square brackets indicate the end value is included, round brackets indicate the end value is not included
- A round bracket is always used with the ∞ symbol

Sets of real numbers may be expressed in a number of ways.

a) Inequality b) Interval Notation c) Graphically (number line)

Ex.

$-2 < x \leq 4$ $(-2, 4]$

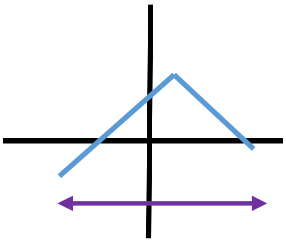


Example: All possible intervals for real numbers a and b, where a < b:

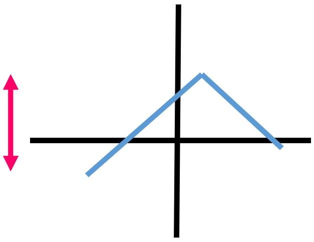
Bracket Interval	Inequality	Number Line	In Words
			The set of all real numbers x such that
(a, b)	$a < x < b$		x is greater than a and less than b
$(a, b]$	$a < x \leq b$		x is greater than a and less than or equal to b
$[a, b)$	$a \leq x < b$		x is greater than or equal to a and less than b
$[a, b]$	$a \leq x \leq b$		x is greater than or equal to a and less than or equal to b
$[a, \infty)$	$x \geq a$		x is greater than or equal to a
$(-\infty, a]$	$x \leq a$		x is less than or equal to a
(a, ∞)	$x > a$		x is greater than a
$(-\infty, a)$	$x < a$		x is less than a
$(-\infty, \infty)$	$-\infty < x < \infty$		x is an element of the real numbers

Domain and Range

The domain of a function is the **set of all first coordinates (x-values)** of the relation.
The range of a function is the **set of all second coordinates (y-values)** of the function.



Domain

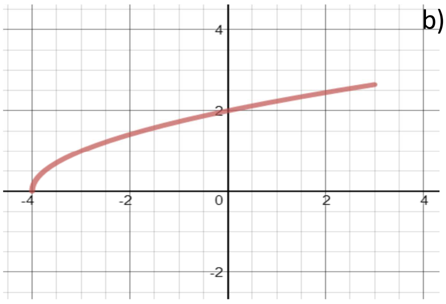


Range

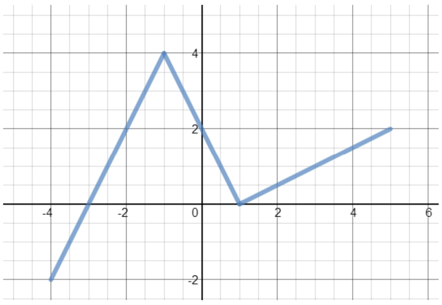
Examples:

1. Given the following relations, state the domain and range.

a)

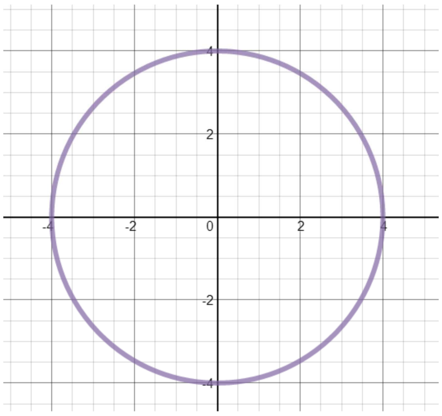


b)

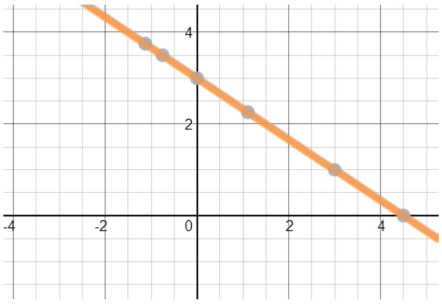


c) {(1, 2), (3, 4), (4, 6), (7, 10)}

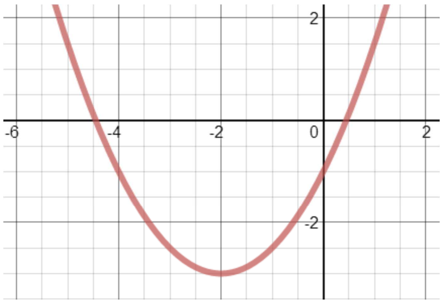
d)



e)



f)



2. Given the equation of the following functions, sketch each function and state their domain and range.

a) $y = x - 5$

b) $y = x^2 + 7$

c) $y = -2(x + 4)^2 + 3$

d) $y = \sqrt{x-3}$

e) $y = \frac{1}{x+3}$

Quadratic Functions

There are 3 forms used to model quadratic functions.

Form	Model	Properties	Example
Standard Form	$y = ax^2 + bx + c$ where a, b, & c are constants and $a \neq 0$	• If $a > 0$, the parabola opens up and has a minimum. • If $a < 0$, the parabola opens down and has a maximum • c is the y-intercept	$y = 3x^2 - 4x + 7$ $a = 3$ and $3 > 0$, so the parabola opens up and has a minimum. 7 is the y-intercept.
Factored Form	$y = a(x – r)(x – s)$ where a, r & s are constants and $a \neq 0$	• If $a > 0$, the parabola opens up and has a minimum. • If $a < 0$, the parabola opens down and has a maximum • Values for r and s are used to find the x-intercepts or zeros.	$y = -2(x + 4)(x – 3)$ $a = -2$ and $-2 < 0$, so the parabola opens down and has a maximum. The x-intercepts are at -4 and 3.
Vertex Form	$y = a(x – p)^2 + q$ where a, p & q are constants and $a \neq 0$	• If $a > 0$, the parabola opens up and has a minimum. • If $a < 0$, the parabola opens down and has a maximum • (p, q) is the vertex	$y = 0.5(x – 3)^2 + 5$ $a = 0.5$ and $0.5 > 0$, so the parabola opens up and has a minimum. The vertex is at (3,5).

Examples: Determine the equation of a quadratic function that satisfies each set of conditions.

a) x-intercepts at -2 and -6, y-intercept at 24.

b) x-intercept at -1, passing through the point (-2, 6).

c) Vertex at (-4, 7) and passing through the point (1, 12).

Factoring Polynomials

Always look for a greatest common factor (GCF) first!

Ex. $8x^3 + 6x^2 = 2x^2(4x + 3)$

If the expression is a binomial, look for a Difference of Squares.

Ex. $x^2 – 25 = (x – 5)(x + 5)$

If the expression is a trinomial in the form $x^2 + bx + c$, look for the Sum (b) and Product (c).

Ex. 1. $x^2 + 9x + 20 = (x + 4)(x + 5)$

Add 9	Mult 20
7	1, 20
-7	2, 10
5	4, 5

If the expression is a trinomial in the form $ax^2 + bx + c$, look for the Sum (b) and Product (a x c), use decomposition approach.

Ex. 2. $2x^2 - 5x + 3 = 2x^2 - 2x - 3x + 3$

$2x(x - 1) - 3(x - 1)$

$(x - 1)(2x - 3)$

Add -5	Mult 6
7	1, 6
-7	-1, -6
5	2, 3
-5	-2, -3

Remember to factor fully where possible.

Ex. 3. $3x^2 - 48 = 3(x^2 - 16)$

$3(x - 4)(x + 4)$

Ex. 4. $2x^3 - 14x^2 + 24x = 2x(x^2 - 7x + 12)$

$2x(x - 4)(x - 3)$

Examples: Factor Fully

a) $3a^4b^2 - 6a^2b^3 + 12ab^4$

b) $36x^2 - 49$

c) $9a^2 - 1$

d) $x^2 - 5x - 14$

e) $6a^2 - 9a - 6$

f) $10y^3 + 5y^2 - 5y^3$

g) $6x^2 - 22x - 40$

h) $4a^2 - 25b^2$

Determining x-intercepts or roots of quadratic functions.

Standard form – factor if possible, set the factors equal to zero.

Ex. $y = x^2 + 10x + 21 = 0$
$y = (x + 3)(x + 7) = 0$

$x = -3$, or $x = -7$

Standard form – factor is not possible, use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (quadratic equation)

Ex. $2x^2 - 4x - 10$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-10)}}{2(2)}$$
$$x = \frac{4 \pm \sqrt{16 + 80}}{4}$$
$$x = \frac{4 \pm \sqrt{96}}{4}$$

$x = 3.45$ or $x = -1.45$

Vertex form – set equation equal to zero, isolate x.

Examples: Determine the x-intercepts, the vertex, the direction of opening, and the domain and range of each quadratic function. Sketch a graph of the function.

a) $y = 2(x - 2)(x + 5)$

b) $y = 3(x - 5)^2 - 9$

c) $y = -3x^2 + 17x + 6$

d) $y = 2x^2 - 12x + 7$

Transformations $y = af(k(x-d)) + c$

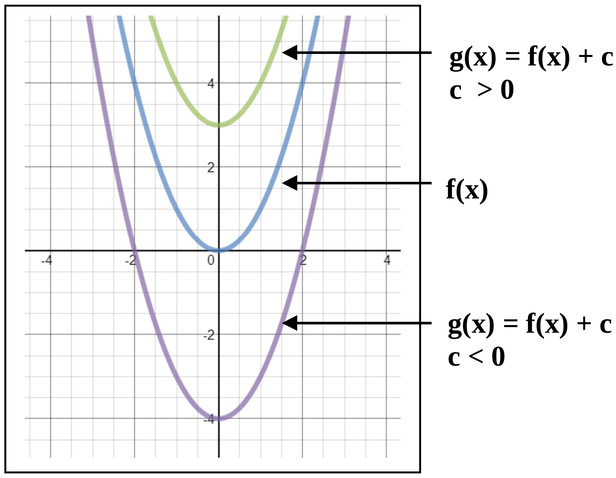
Translations

A transformation that results in a shift of the original figure without changing its shape.

Vertical Translation of **c** units:

The graph of the function **g(x) = f(x) + c**

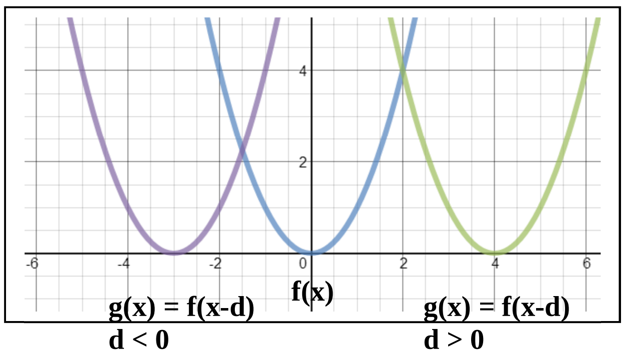
- when c is positive, the translation is UP by c units.
- when c is negative, the translation is DOWN by c units.



Horizontal Translation of **d** units:

The graph of the function **g(x) = f(x - d)**

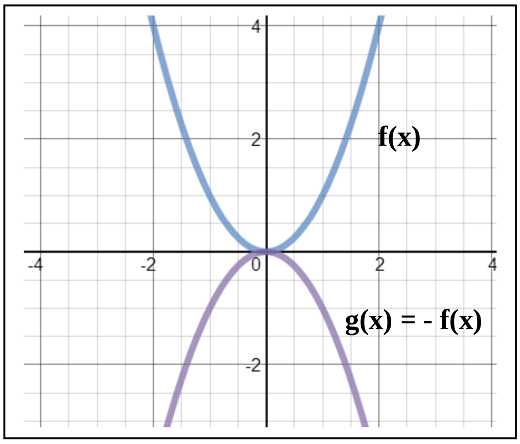
- when d > 0, the translation is to the RIGHT by d units.
- when d < 0, the translation is to the LEFT by d units.



Reflections
A transformation in which a figure is reflected over a reflection line.

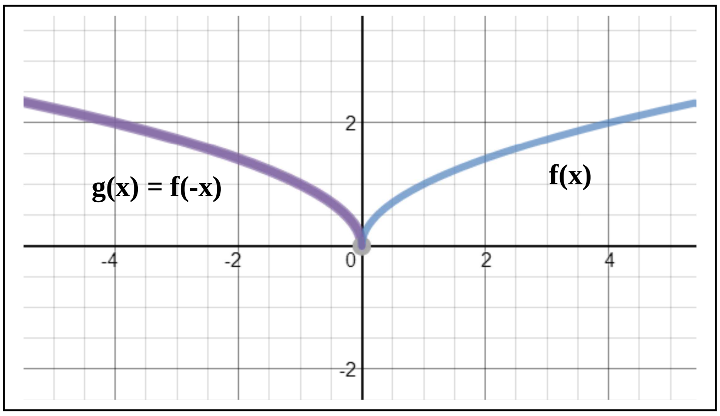
Reflection in the X-axis/Vertical Reflection:

The graph of g(x) = -f(x)



Reflection in the Y-axis/Horizontal Reflection:

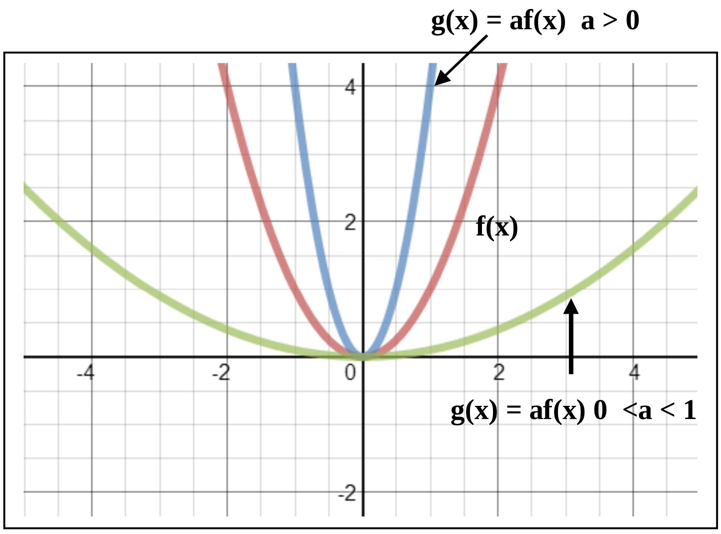
The graph of g(x) = f(-x)



Vertical Stretch and Compression:

The graph of the function g(x) = af(x), a>0

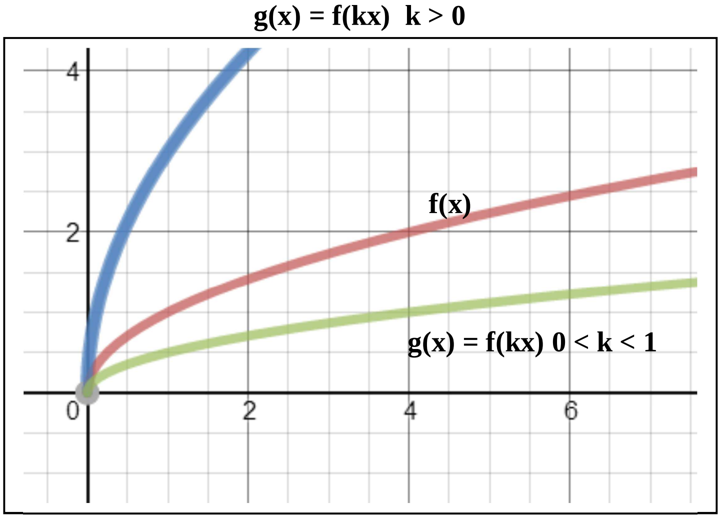
- when |a|>1, there is a VERTICAL STRETCH by a factor of a.
- when 0<|a|<1, there is a VERTICAL COMPRESSION by a factor of a.
- Points on the x-axis are invariant.



Horizontal Stretch and Compression:

The graph of the function g(x) = f(kx), k > 0

- when |1/k| >1, there is an EXPANSION (STRETCH) by a factor of 1/k.
- when 0 < |k| <1, there is a COMPRESSION by a factor of 1/k.



Examples: Identify each transformation of the function y = f(x).

a) y = 2f(x) + 1

b) $y = -\frac{1}{3}f(x-2)$

c) y = f(-3x)

d) y = -2 f(3x + 3) - 4

Examples: Write an equation for the transformed function of each base function. State the domain and range of each.

a) f(x) = x^2, is reflected in the x-axis, stretched vertically by a factor of 3, translated to the left 6 units and down 5 units.

b) $f(x) = \sqrt{x}$, is compressed horizontally by a factor of 0.5, stretched vertically by a factor of 3, reflected in the y-axis, and translated right 4 units.

Section 1.1 Power Functions

A **power function** is the simplest type of **polynomial function** and has the form **f(x) = ax^n**, where **x** is a variable, **a** is a real number and **n** is a whole number.

A **polynomial expression** is an expression of the form:

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + . . . + a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0$$

Recall, we very seldom show exponent values of 1, and $x^0 = 1$

so $$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + . . . + a_3x^3 + a_2x^2 + a_1x + a_0$$

Where

- *n* is a whole number
- *x* is a variable
- the coefficients a_0, a_1, a_2, ... are real numbers
- the **degree** of the expression is *n*, the exponent on the greatest power of x
- a_n, is the coefficient of the greatest power of x, and is called **the leading coefficient**
- a_0, the term without a variable, is the **constant term**

A **polynomial function** has the form:

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + . . . + a_3x^3 + a_2x^2 + a_1x + a_0$$

Traditionally, polynomial functions are written in descending order of powers of **x**.
(It keeps things looking nice and neat)

P.S the exponents in the function do not need to decrease consecutively, some terms may have zero as the coefficient. So $f(x) = 12x^4 + 2x^2 + 5$ is still a polynomial function, it just means that for ease we did not show the zero coefficient terms $f(x) = 12x^4 + 0x^3 + 2x^2 + 0x + 5$.

Some **Power Functions** have special names that are associated with their **degree**

Power Function	Degree	Name
y = a	0	Constant
y = ax	1	Linear
$y = ax^2$	2	Quadratic
$y = ax^3$	3	Cubic
$y = ax^4$	4	Quartic
$y = ax^5$	5	Quintic

Example: Determine which functions are polynomials. Justify your answer.
State the degree and the leading coefficient of each polynomial function.

a) $g(x) = \cos x$

b) $f(x) = 3x^4$

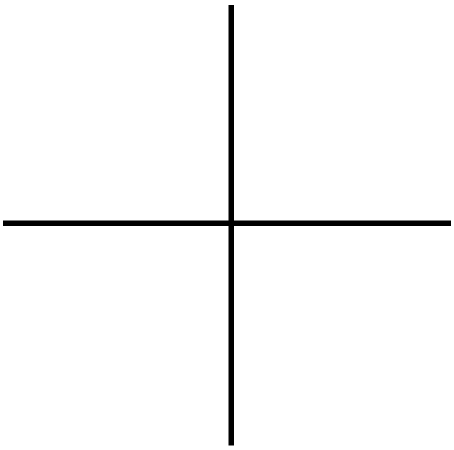
c) $f(x) = x^5 - 3x^3 + 7x^2 - x + 1$

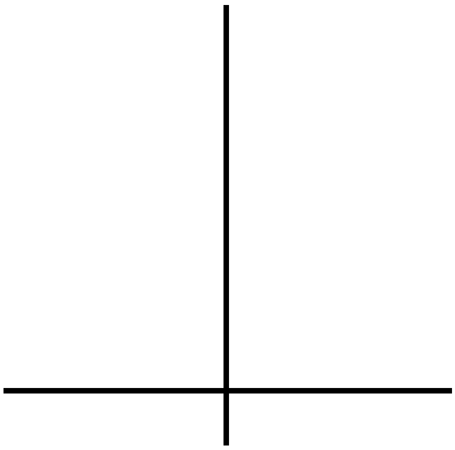
d) $h(x) = 4^x$

Investigate Power Functions

Graph the following using available technology. Make a sketch with labels.

$y = x$, $y = x^3$, $y = x^5$, $y = x^7$ and $y = x^2$, $y = x^4$, $y = x^6$, $y = x^8$





Power functions have similar characteristics depending on whether their degree is even or odd.

Odd Degree Power Functions: Graphs that curve from quadrant 3 to quadrant 1. The higher the exponent the closer the curve gets to the y-axis.

Even Degree Power Functions: Graphs that make a U-shape. The higher the exponent the U shape gets closer to the y-axis.

End behaviour: The end behaviour of a function is the behaviour of the **y-values** as x increases (that is, as x approaches positive infinity, $x \to \infty$) and as x decreases (that is, as x approaches negative infinity, $x \to -\infty$)

Example: Write each of the following power functions in the appropriate row of the second column of the table below. Give reasons for your choices.

$y = 2x$ $\quad y = 5x^6$ $\quad y = -3x^2$ $\quad y = x^7$ $\quad y = -\frac{2}{5}x^9$

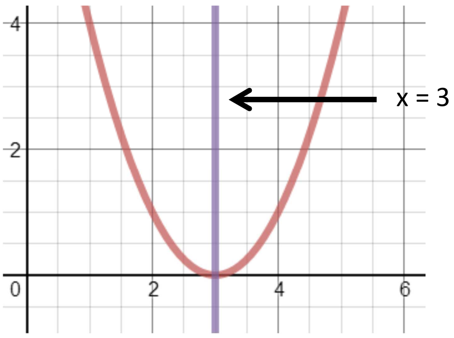
$y = -4x^5$ $\quad y = x^{10}$ $\quad y = -0.5x^8$

End Behaviour	Function	Reasons
Extends from quad 3 to quad 1		
Extends from quad 2 to quad 4		
Extends from quad 2 to quad 1		
Extends from quad 3 to quad 4		

Line Symmetry

A graph has line symmetry if there is a line **x = a** that divides the graph into two equal parts such that one part is a reflection of the other in the line **x = a**.

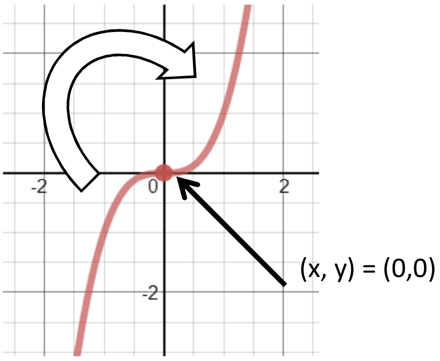
- Even-degree power functions have line symmetry.



Point Symmetry

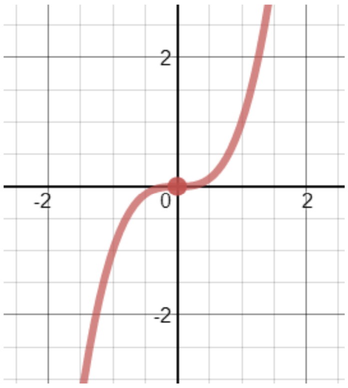
A graph has point symmetry about a point (a, b) if each part of the graph on one side of (a, b) can be rotated 180° to coincide with part of the graph on the other side of (a, b).

- Odd-degree power functions have a point of symmetry.

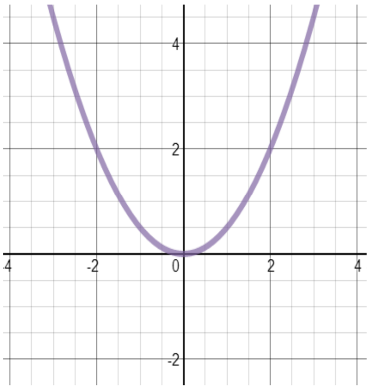


Example: For each of the following functions, state the domain and range, describe the end behaviour and identify any symmetry.

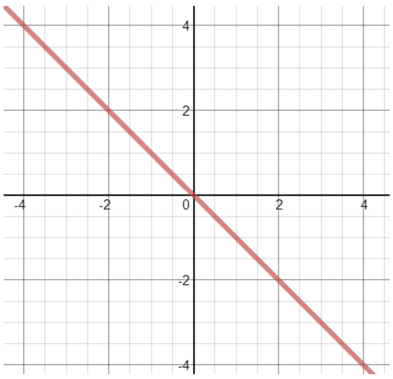
a)



b)



c)



Example: The volume of a helium balloon is given by the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius of the balloon, in meters and $r \in [0, 5]$

a) Sketch V(r).

b) State the domain and range in this situation.

c) Describe the similarities and difference between the graph of V(r) and the graph of $f(x) = x^3$

Section 1.2 Characteristics of Polynomial Functions

In section 1.1 we explored Power Functions, a single piece of a polynomial function. This modelling method works perfectly for simple real world problems such as:

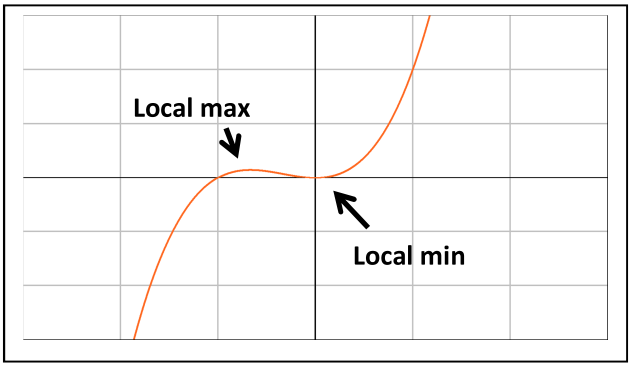
- Area Square $\rightarrow$ $A(x) = x^2$
- Volume Cube $\rightarrow$ $V(x) = x^3$
- Area Circle $\rightarrow$ $A(r) = \pi r^2$
- Volume Sphere $\rightarrow$ $V(r) = \frac{4}{3}\pi r^3$

But the more complex the situation, the more complex the function required. For example, a patient's response time to certain medication is modelled using a slightly more complex polynomial function $r(d) = -0.7d^3 + d^2$ where $r(d)$ is the reaction time in seconds, and d is the dosage of medication administered.

When combining power functions into a single polynomial function, there are a few new features we like to look for, such as

Local Minimum and Maximum Points:

Let's look at the graph of the polynomial function defined by $f(x) = x^3 + x^2$

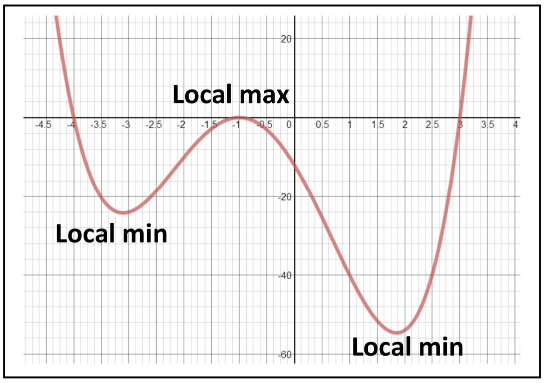


Looks more like $f(x) = x^3$ than $f(x) = x^2$, but there is a small change we now have "bumps" on the graph

In general, polynomial function graphs consist of a smooth curve with a series of hills and valleys. The hills and valleys are called **turning points**. Each turning point corresponds to a **local maximum** or **local minimum** point.

Let's look at a more complex polynomial function defined by

$f(x) = x^4 + 3x^3 - 9x^2 - 23x - 12$



** **The maximum possible number of local min/max points is one less than the degree of the polynomial**.**

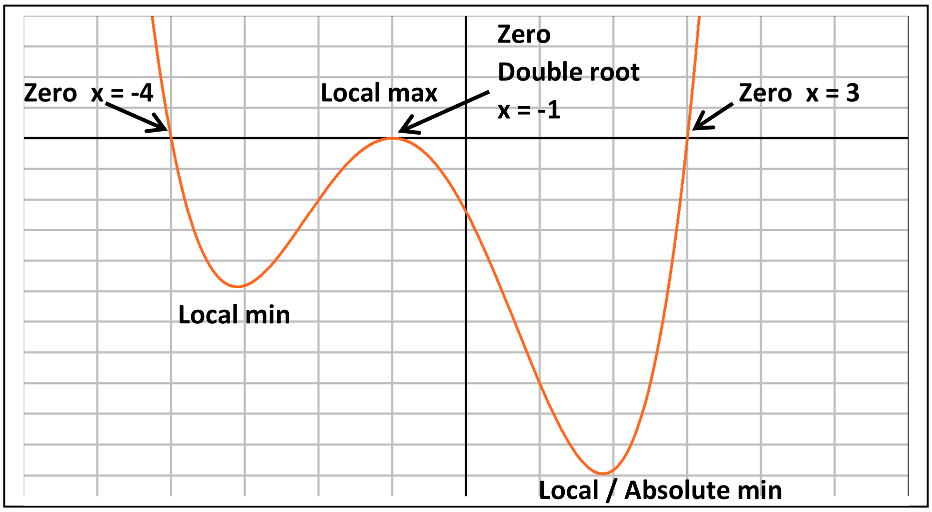
Example: The polynomial above has degree 4 and has two local minimums and one local maximum for a total of three. This is the maximum possible number of local minimum and maximum points for a polynomial of this degree.

Zeros (or x-intercepts) of polynomial functions:

A zero of a polynomial function is an x-value for which y = 0. At these x-values, its graph intersects or touches the x-axis.

** **The *maximum* number of zeros of any polynomial is the same as its degree, there may be less depending on the nature of the function and the possibility of repeated roots****

Example: The polynomial function $f(x) = x^4 + 3x^3 - 9x^2 - 23x - 12$, is shown below and only has only three zeros, not four. This is one less than the maximum of four zeros that a polynomial of degree four can have. This polynomial intersects the x-axis at x = -4 and 3, but only touches the x-axis at x = -1.



Finite Differences: (used to find leading terms and determine degree from a table of values**)**

Example:

Recall for linear functions $f(x) = 3x + 2$ we could make a table of values

x	y	1st Diff
0	2	
		3
1	5	
		3
2	8	
		3
3	11	
		3
4	14	
		3
5	17	

First Difference is constant, so degree is equal to 1 and leading coefficient is 3

Example:

Recall for quadratic functions $f(x) = 3x^2 + 2x + 1$ we could make a table of values

x	y	1st Diff	2nd Diff
0	1		
		5	
1	6		6
		11	
2	17		6
		17	
3	34		6
		23	
4	57		6
		29	
5	86		

Second Difference is constant, so degree is equal to 2 but the leading coefficient is not 6 it should be 3. So how do we account for this?

For a polynomial of degree n, where n is a positive integer, the nth differences

- are constant (equal)
- have the same sign as the leading coefficient
- are equal to a(n!), where a is the leading coefficient

Factorial (!) means: n! = n(n - 1)(n - 2)(n - 3) ... (2)(1)

5! = 5(4)(3)(2)(1)
= 120

So for our example above the second difference is constant, so degree is equal to 2 but the leading coefficient is a(n!).

6 = a(2!) because n = 2 (2nd difference is where we found the constant value)
6 = a(2)(1)
6 = 2a
3 = a

Example: Each table of values represents a polynomial function. Use finite differences to determine

i) the degree of the polynomial function
ii) the sign of the leading coefficient
iii) the value of the leading coefficient

a)

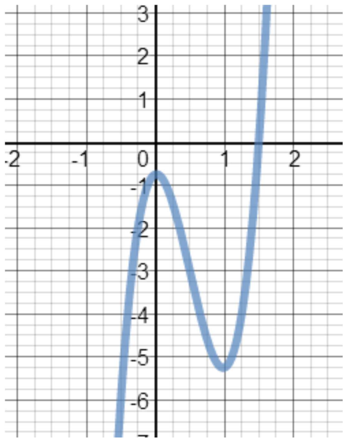
x	y
0	4
1	-1
2	-12
3	-29
4	-52
5	-81
6	-116
7	-157
8	-204

b)

x	y
0	1
1	5
2	14
3	30
4	55
5	91
6	140
7	204
8	285

Key Features of Graphs of Polynomial Functions with Odd Degree

- Odd-degree polynomials have at least one zero, up to a maximum of n x-intercepts, where n is the degree of the function.
- The domain is $\{x \in R\}$ and the range is $\{y \in R\}$.
- They have no absolute maximum point and no absolute minimum point.
- They may have point symmetry.



Positive Leading Coefficient

- Graph extends from quadrant 3 to quadrant 1.
  OR "as $x \to -\infty$ $y \to -\infty$" and "as $x \to \infty$, $y \to \infty$"

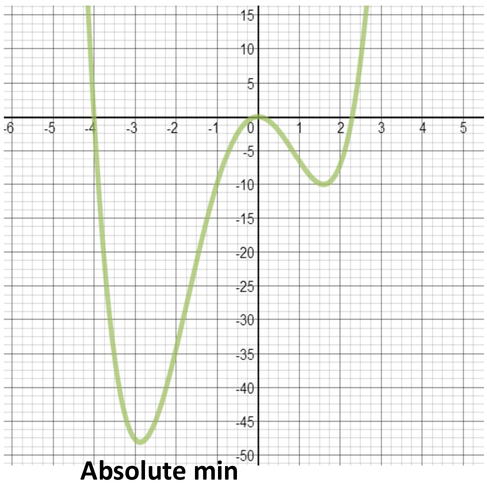
Negative Leading Coefficient

- Graph extends from quadrant 2 to quadrant 4.
  OR "as $x \to -\infty$, $y \to \infty$" and "as $x \to \infty$, $y \to -\infty$"

Key Features of Graphs of Polynomial Functions with Even Degree

- Even-degree polynomials may have no zeros, up to a maximum of n x-intercepts, where n is the degree of the function.
- The domain is $\{x \in R\}$.
- They may have line symmetry.

Positive Leading Coefficient



- Graph extends from quadrant 2 to quadrant 1.
  OR "as $x \to -\infty$, $y \to \infty$" and "as $x \to \infty$, $y \to \infty$"
- The range is $\{y \in R \mid y \geq a\}$, where a is the absolute minimum value of the function.
- It will have at least one minimum point.

Negative Leading Coefficient

- Graph extends from quadrant 3 to quadrant 4.
  OR "as $x \to -\infty$, $y \to -\infty$" and "as $x \to \infty$, $y \to -\infty$"
- The range is $\{y \in R \mid y \leq a\}$, where a is the absolute maximum value of the function.
- It will have at least one maximum point.

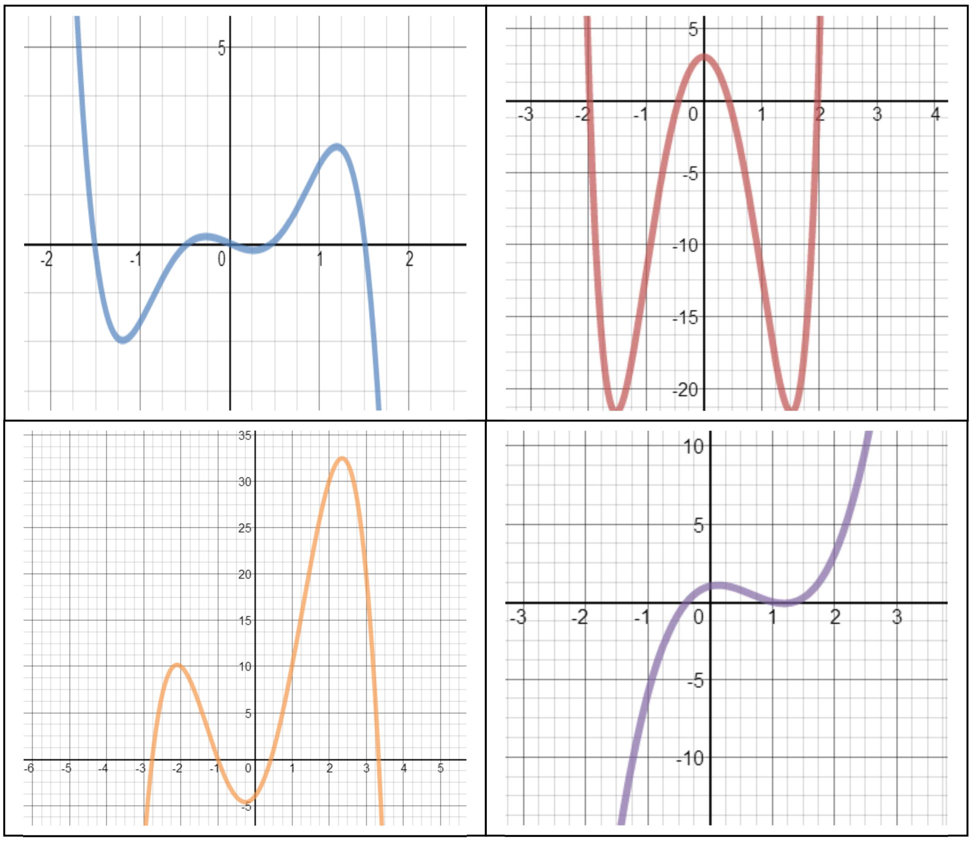
Example: Determine the key features of the graph of each polynomial. Use these key features to match each function with its graph. State the number of local maximum and minimum points for the graph of each function.

a) $f(x) = 2x^3 - 4x^2 + x + 1$

b) $f(x) = -x^4 + 10x^2 + 5x - 4$

c) $f(x) = -2x^5 + 5x^3 - x$

d) $f(x) = x^6 - 16x^2 + 3$



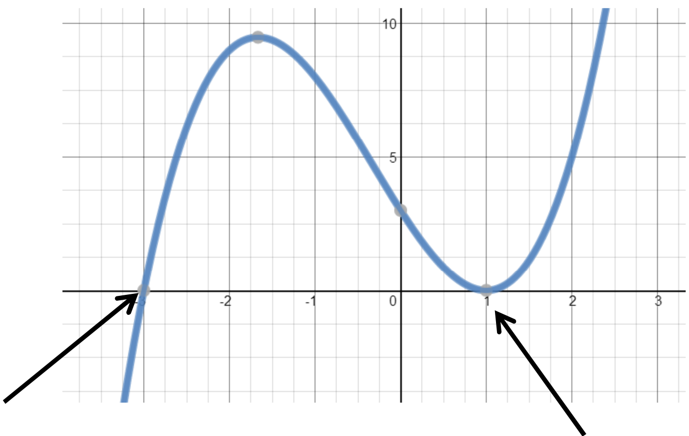
Section 1.3 Equations and Graphs of Polynomial Functions

The graph of a polynomial function can be sketched using the **x-intercepts**, the **degree** of the function, and the **sign of the leading coefficient**.

- The x-intercepts of the graph of a polynomial function are the roots of the corresponding polynomial equation.
- When a polynomial function is in factored form, the zeros can be easily determined from the factors. When a factor is repeated n times, the corresponding zero has order n.

Example: The function $f(x) = (x - 1)^2(x + 3)$ has zeros at 1 and -3. However since the factor $(x - 1)$ is repeated twice, $x = 1$ is a zero of order 2.

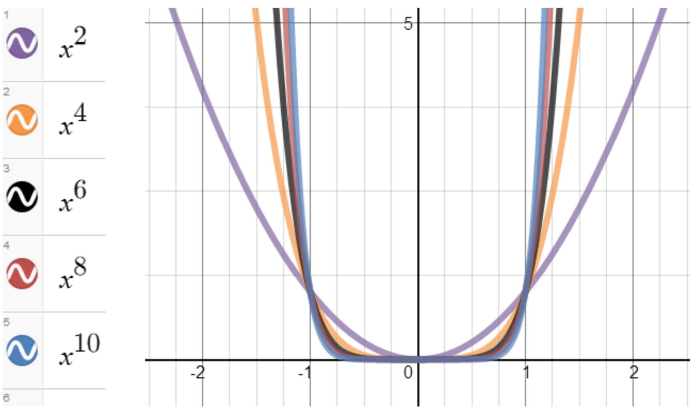
- The graph of a polynomial function changes sign (y coordinates change sign) only at zeros of odd order. At zeros of even order, the graph touches but does not cross the x-axis.



x= - 3 has order 1
The sign of the function changes
(the function will cross the x-axis)

x= 1 has order 2
The sign of the function does not change
(the function will not cross the x-axis)

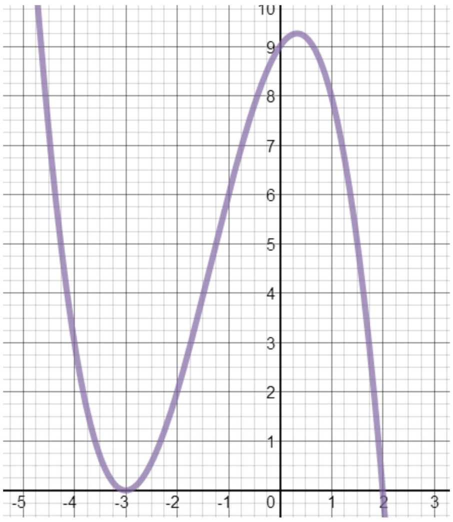
** The higher the even order number, the flatter the graph will be near the x-axis. **



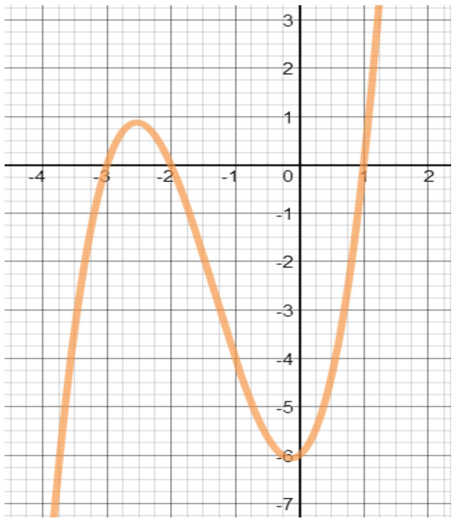
Example: For the following graphs of polynomial functions, determine

a) the least possible degree and sign of the leading coefficient
b) the x-intercepts and the factors of the function
c) the intervals where the function is positive and the intervals where it is negative
d) an equation for the polynomial function that corresponds to the graph

i)



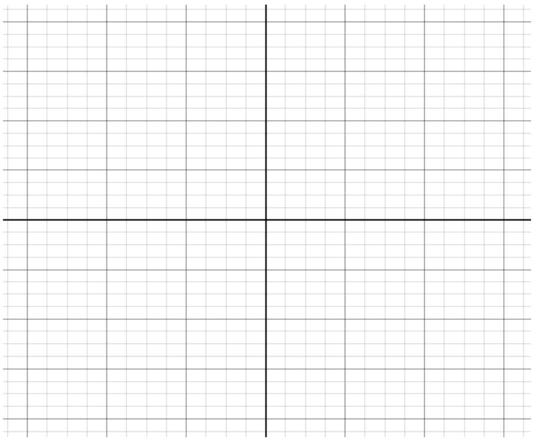
ii)



Example: Sketch a graph of the following polynomial functions defined by

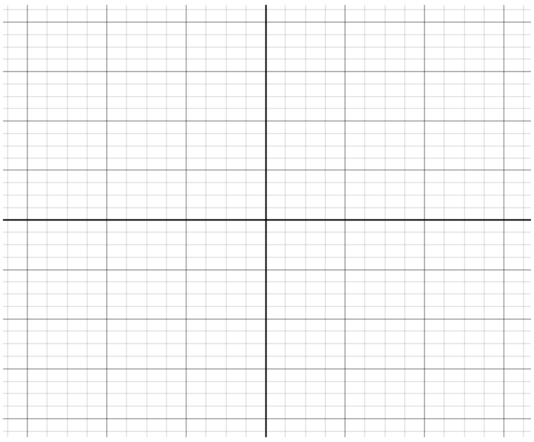
a) $y = -2(x + 1)^2(x - 2)$

Degree	Leading Coefficient	End Behaviour	Zeroes	y- intercept



b) $y = (x - 1)^2(x + 2)(x + 4)$

Degree	Leading Coefficient	End Behaviour	Zeroes	y- intercept



Even Functions

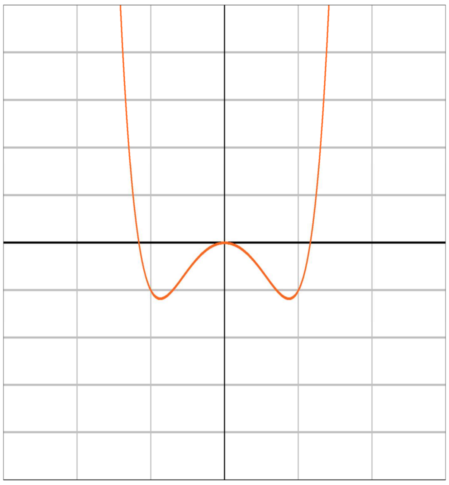
An even degree polynomial function is an "even function" if:

i) the exponent of each term is even.
ii) the function satisfies the property f(-x)= f(x) for all x in its domain
iii) the function is symmetric about the y-axis.

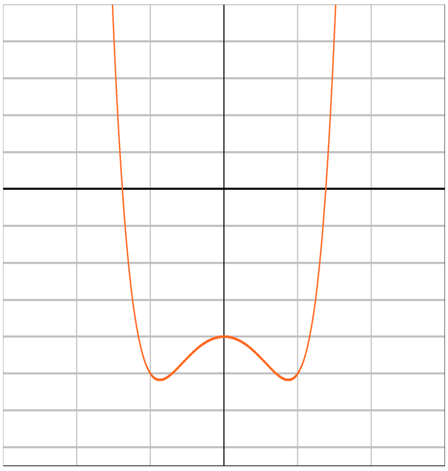
Example: $f(x) = 2x^6 + x^4 - 5x^2$

Constant

Example: $f(x) = 2x^6 + x^4 - 5x^2 - 8$



Even Function



Still Even Function

f(-x)= f(x) means that the reflection over the y-axis is the same as the original

*When a constant is added to an even function, the function remains even. The constant causes a vertical translation, and does not affect symmetry about the y-axis. **

Odd Functions

An odd-degree polynomial function is an odd function if:

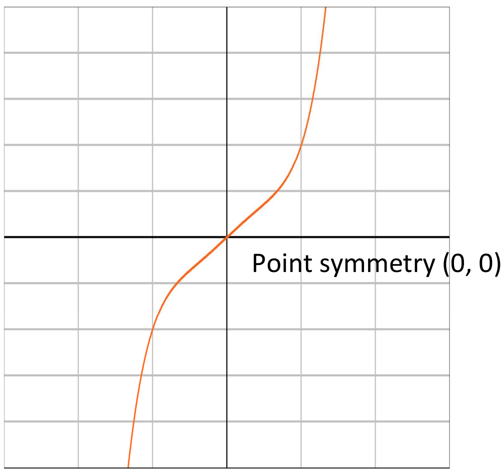
i) the exponent of each term is odd.

ii) the function satisfies the property f(-x) = - f(x) for all x in its domain

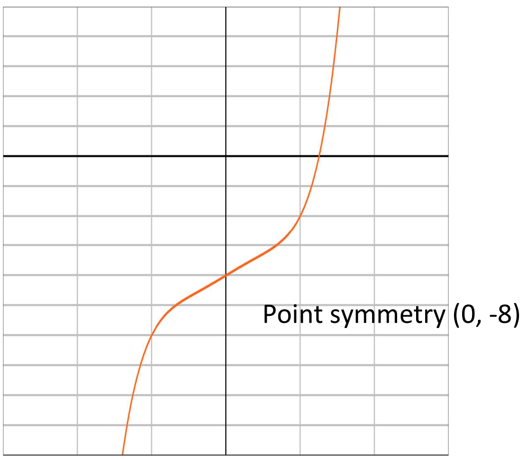
iii) the function is rotationally symmetric about the origin.

Constant

Example: $f(x) = 2x^5 - x^3 + 3x$ Example: $f(x) = 2x^5 - x^3 + 3x - 8$



Odd Function



No longer an Odd Function

f(-x)= - f(x) means that a reflection over the y-axis followed by a reflection over the x-axis is the same as the original (180° rotation about the origin)

*When a constant is added to an odd function, the function does not remain odd. The constant again causes a vertical translation, and does affect point symmetry about the origin. **

Example: Without graphing, determine if each polynomial function has line symmetry, point symmetry, or neither. Justify your answer.

a) $f(x) = -5x^4 + 3x^2 - 4$

b) $g(x) = x(2x + 3)(x - 2)$

Section 1.4 Transformations

Transformations

The graph of a function of the form $f(x) = a[k(x-d)]^n + c$ is obtained by applying the transformations to the graph of the power function $f(x) = x^n$, where n is a non-negative integer, $n \in N$.

The parameters **a, k, d**, and **c** correspond to the following transformations:

- **a** corresponds to a vertical stretch (if **a** > 1) or compression (if 0 < **a** < 1) by a factor of **a**
  if **a** < 0, a reflection in the x-axis.

- **k** corresponds to a horizontal stretch (if 0 < **k** < 1) or compression (if **k** > 1) by a factor of 1/**k**
  if **k** < 0, a reflection in the y-axis.

- **c** corresponds to a vertical translation up (if **c** > 0) or down (if **c** < 0)

- **d** corresponds to a horizontal translation to the left (if **d** < 0) or right (if **d** > 0)

An accurate sketch of the transformed graph is obtained by applying transformations represented by **a** and **k** before the transformations represented by **c** and **d**.

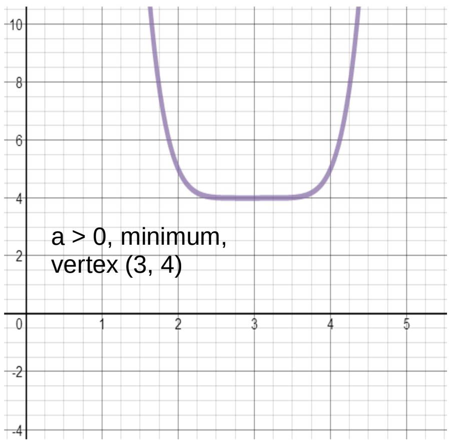
R.S.T = **R**eflect **S**tretch(Compress) **T**ranslate

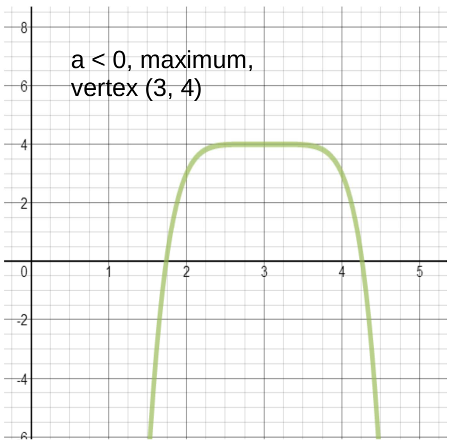
Even-Degree Polynomial Functions

When **n is even**, the graph of the polynomial function $f(x) = a[k(x - d)]^n + c$ is an even-degree function and has a vertex at (**d, c**). The axis of symmetry is x = d.

- For **a** > 0, the graph opens upward. Therefore the graph extends from Q2 to Q1.
- The vertex is the minimum point on the graph and **c** is the minimum value. The range of the function is $\{y \in R \mid y \geq \boldsymbol{c}\}$.

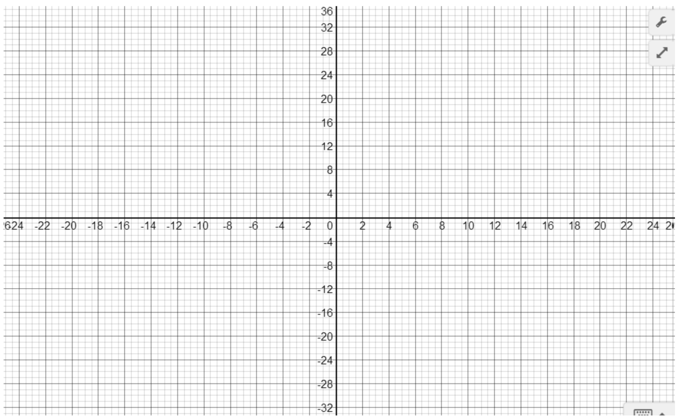
- For **a** < 0, the graph opens downward. Therefore the graph extends from Q3 to Q4.
- The vertex is the maximum point on the graph and **c** is the maximum value. The range of the function is $\{y \in R \mid y \leq \boldsymbol{c}\}$.





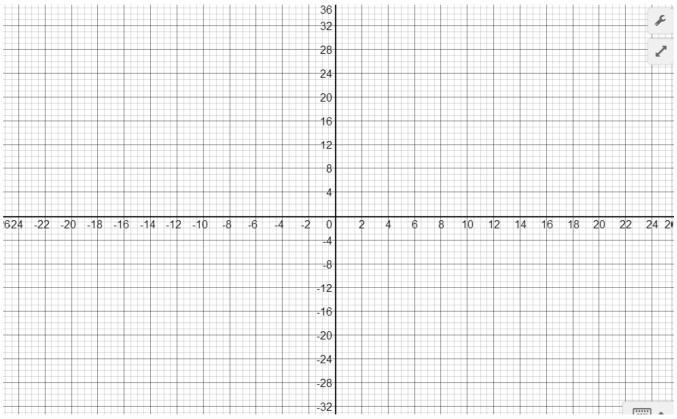
Example: Given a base function of $f(x) = x^4$, state the parameters and the corresponding transformations required to obtain the graph of $g(x) = -2\left[\frac{1}{3}(x-4)\right]^4 - 1$

Sketch the graph and state the domain, range, vertex, and equation of the axis of symmetry for the transformed function.



Example: Given a base function of $f(x) = x^3$, state the parameters and the corresponding transformations required to obtain the graph of $g(x) = 3[-2(x+1)]^3 + 5$

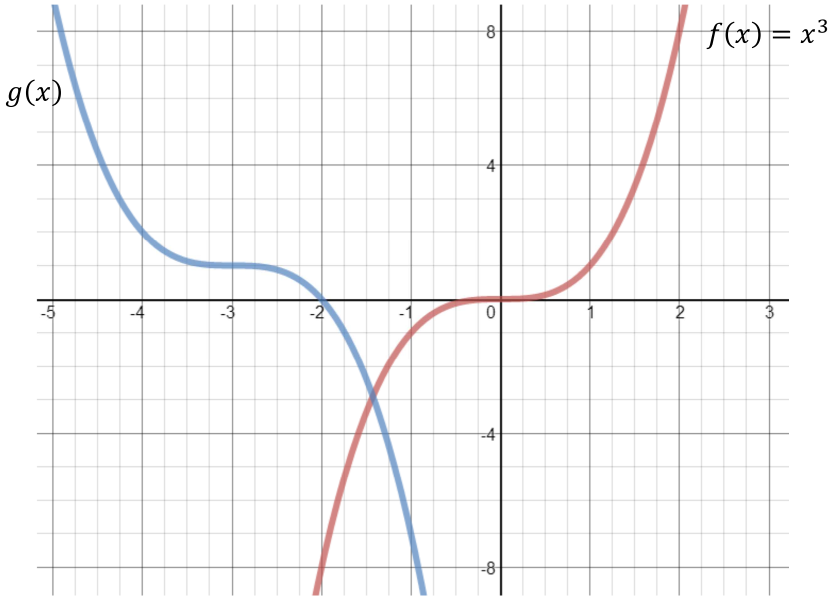
Sketch the graph and state the domain, range, vertex, and equation of the axis of symmetry for the transformed function.



Example: Describe the transformations that must be applied to the graph of each power function, $f(x)$, to obtain the transformed function. Then, write the corresponding equation of the transformed function. State the domain and range for each transformed function, and state the vertex and equation of the axis of symmetry for any even functions.

a) $f(x) = x^5, g(x) = -\frac{1}{2}f(3x-6)$ b) $f(x) = x^6, g(x) = \frac{4}{3}f\left[-\frac{1}{3}(x+5)\right] - 1$

Example: Transformations are applied to the power function $f(x) = x^3$ to obtain the resulting graph. Determine an equation for the transformed function, g(x). State its domain and range.



Example: Write an equation for the function that would result from the given transformations. Then state the domain and range, the vertex, and the equation of the axis of symmetry. The function $f(x) = x^4$ is reflected in the x-axis, stretched vertically by a factor of 3.5, compressed horizontally by a factor of 2/3, translated 6 units to the right, and 7 units down.

Section 1.5 Slopes of Secants and Average Rate of Change

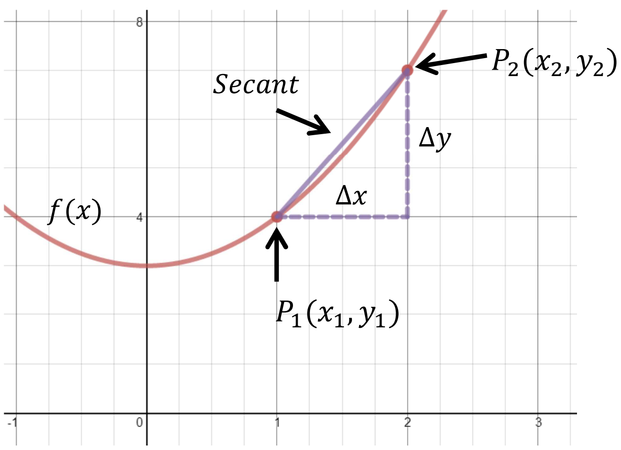
Key Concepts

- A secant is a straight line that connects two points on a curve.
- Rate of Change (Slope) is a measure of how quickly one quantity (the dependent variable) changes with respect to another quantity (the independent variable).

There are two types of rates of change, average and instantaneous.

Average rates of change

- represent the rate of change over a specified interval corresponding to the slope of a secant between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a curve

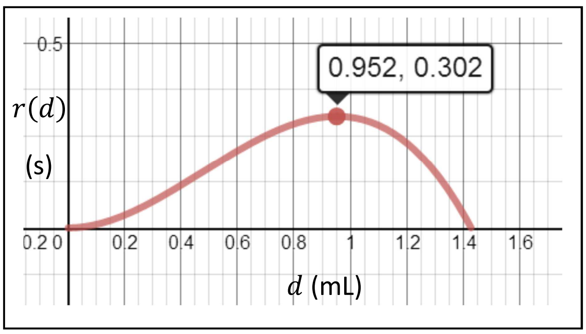


$$\text{Avg. Rate of Change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- An average rate of change can be determined by calculating the slope between two points given in a table of values or by using an equation.

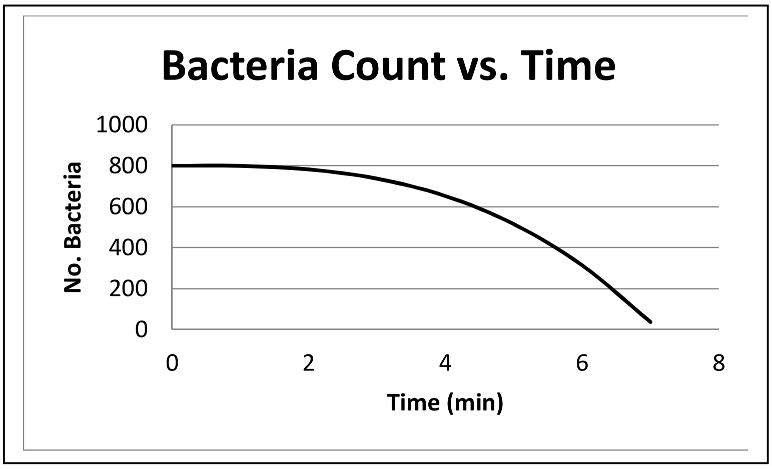
Example: A medical researcher establishes that a patient's reaction time, r, in minutes, to a dose of a particular drug is $r(d) = -0.7d^3 + d^2$, where d is the amount of the drug, in mL, that is absorbed into the patient's blood, and $d \in [0, 1.428]$.

Use the graph to describe when the rate of change is positive, when it is zero, and when it is negative.



Example: A new antibacterial spray is tested on a bacterial culture. The table shows the population, P, of the bacterial culture t minutes after the spray is applied.

t (min)	P
0	800
1	799
2	782
3	737
4	652
5	515
6	314
7	37



How can you tell the average rate of change is negative by examining

i) the table of values

ii) the graph

Determine the average rate of change of the number of bacteria over the entire time period shown in the table. Interpret this value for this situation.

Compare the average rates of change of the number of bacteria in the first 3 minutes and the last 3 minutes. Explain any similarities and differences. Draw the secants on the graph and label them AB and CD respectively.

Example: A football is kicked into the air such that its height, h, in metres after t seconds is modeled by the function $h(t) = -4.9t^2 + 14t + 1$. Determine the average rate of change of the height of the ball for each time interval. Interpret these values.

a) [0, 0.5]

b) [2, 2.5]

Section 1.6 Slopes of Tangents and Instantaneous Rate of Change

- A tangent to a curve is a line that intersects a curve at exactly one point.
- An instantaneous rate of change corresponds to the slope of a tangent at a point on a curve.
- An approximate value for an instantaneous rate of change at a point may be determined using....

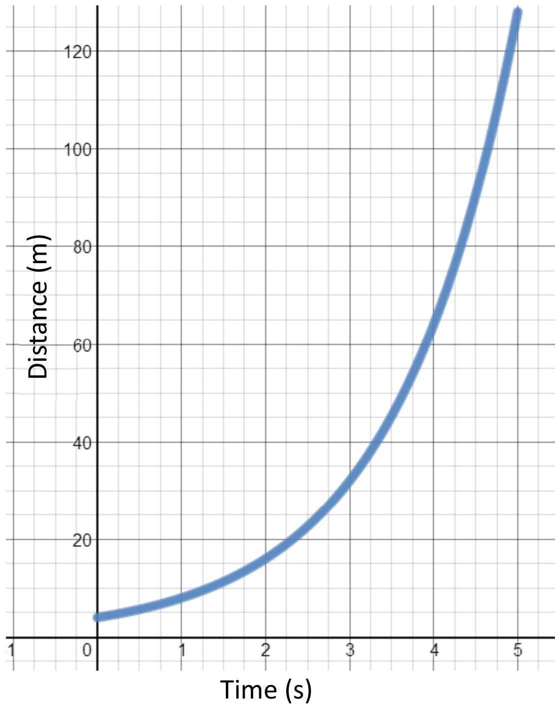
1. a **graph**, either by estimating the slope of a secant passing through that point OR by sketching the tangent and estimating the slope between the tangent point and a second point on the approximate tangent line.
2. a **table of values**, by estimating the slope between the point and a nearby point in the table.
3. an **equation**, by estimating the slope using a very short interval between the tangent point and a second point found using the equation.

Relationship Between the Slope of Secants and the Slope of a Tangent

- As a point Q becomes very close to a tangent point P, the slope of the secant line becomes closer to (approaches) the slope of the tangent line.
- Often an arrow is used to denote the word "approaches". So, the above statement may be written as follows:
  As Q → P, the slope of the secant PQ → the slope of the tangent at P.
- Thus, the average rate of change between P and Q becomes closer to the value of the instantaneous rate of change at P.

Example: The graph shows the approximate distance travelled by a parachutist in the first 5 seconds after jumping out of a helicopter. How fast was the parachutist travelling 2s after jumping out of the helicopter?

i) Use the slope of a secant.
ii) Use 2 points on an approximate tangent.



Example: In the table below, the distance of the parachutist in the previous example is recorded at 0.5 second intervals. Estimate the parachutist's velocity at 2 sec.

Time (s)	Distance (m)
0	0
0.5	1.25
1	5
1.5	11.25
2	20
2.5	31.25
3	45
3.5	61.25
4	80

Example: The functions $d(t) = 5t^2$ can be used to approximate the distance travelled by the parachutist in the previous two examples. Use the equation to estimate the velocity of the parachutist after 2 seconds.

Interval	Δd	Δt	$\Delta d / \Delta t$