Introduction to vectors

A **vector** is a quantity that has both magnitude and direction.

A **scalar** is a quantity that can be described by a single number such as magnitude or size.

Example

Speed is a scalar quantity. The car is travelling at 90 km/hr.

Distance is a scalar quantity. Brianna lives 60 km from Oshawa.

Mass is a scalar quantity. Mr. Lieb has a mass of 95 kg.

Velocity is speed in a given direction. This is a vector quantity. The velocity of the car is 90 km/hr North.

Displacement is distance travelled in a given direction. This is a vector quantity. Brianna lives 60 km Northwest of Oshawa.

Weight is force downwards due to gravity. It is a vector quantity. Mr Lieb has a weight of 925 N (downwards).

**(NOTE) N is the symbol for Newton's which is the metric unit of force measurement

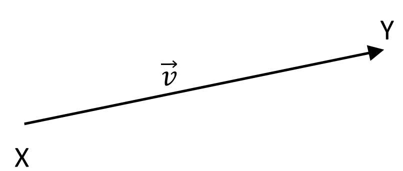
Notation of Vectors

Arrows are used to represent vectors. The pointed portion of the vector is the **head** and the other end is called the **tail**.

The arrowhead points in the direction of the vector and the length of the arrow represents the magnitude of the vector.

The arrows are usually drawn to scale so that the length and direction of the line segment accurately represent the magnitude and direction of the vector.

A vector from point X to point Y is written $\overrightarrow{XY}$, X is the tail or the starting point and Y is the head or the finishing point. This vector could also be denoted with a single lowercase letter such as $\vec{v}$.



The magnitude of a vector is written as $\left|\overrightarrow{XY}\right|$, $|\vec{v}|$. The absolute bars for the magnitude are present because it is not possible to have negative length.

Example

Draw vectors to represent each of the following.

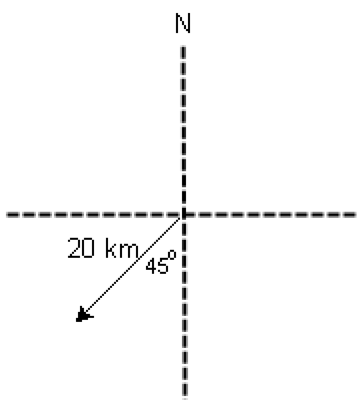
a. displacement of 20 km southwest

b. a weight of 40 N vertically downward

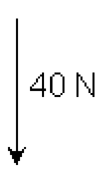
c. a velocity of 100 km/hr on a bearing of $300°$

Solution

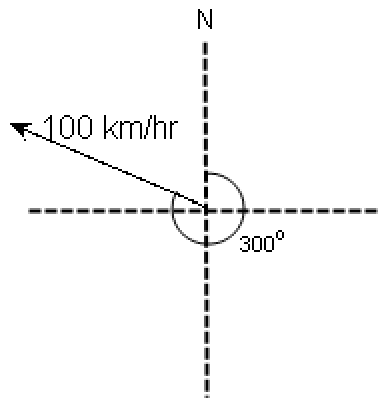
a. Choose an appropriate scale: 1 cm: 10 km. Remembering and assuming that North would be represented at the top then use a protractor to measure 45 degrees between south and west. Draw the arrowhead 2 cm from the initial point and label.



b.
1 cm : 20 N



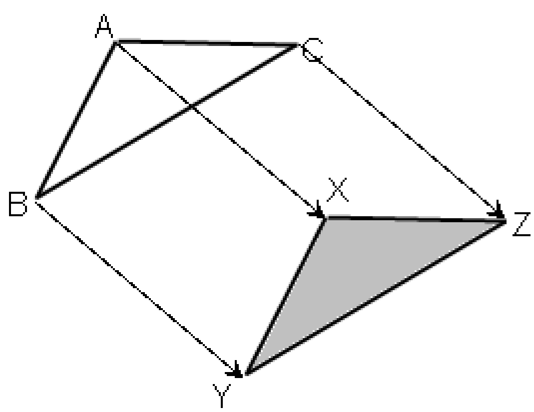
c.
1 cm : 75 km/h



Equal Vectors

Equal vectors do not have to have the same location in space. They DO NOT have to have the same starting and finishing point.

When a figure is translated, each point on that figure moves the same distance and in the same direction. The vectors associated with that translation have the same direction and magnitude and are therefore equal vectors.

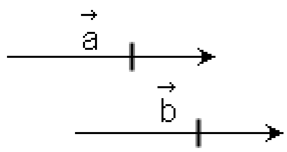


Vectors $\overrightarrow{AX}$, $\overrightarrow{BY}$, and $\overrightarrow{CZ}$ are equal vectors

Equal Vectors

Equal vectors have the same magnitude and direction.

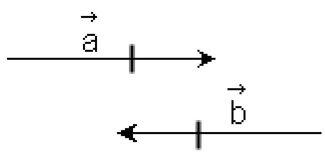
The vectors $\vec{a}$ and $\vec{b}$ below are equal since $|\vec{a}| = |\vec{b}|$ and the direction of $\vec{a}$ is the same as the direction of $\vec{b}$. We write $\vec{a} = \vec{b}$



Opposite Vectors

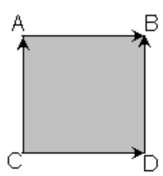
Opposite vectors have the same magnitude, but have an opposite direction.

The vectors $\vec{a}$ and $\vec{b}$ below are opposites since $|\vec{a}| = |\vec{b}|$ and the direction of $\vec{a}$ is the opposite as the direction of $\vec{b}$. We write $\vec{a} = -\vec{b}$



Example

Below is a diagram of a square. List 1 pair of equal and one pair of opposite vectors.



Solution

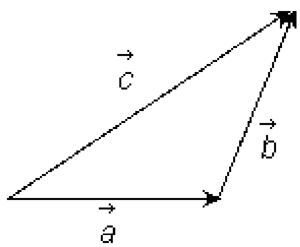
Since $\overrightarrow{AB}$ and $\overrightarrow{CD}$ have the same direction and are the same length then these are one of two pairs of equal vectors. Another is $\overrightarrow{CA}$ and $\overrightarrow{DB}$.

Since $\overrightarrow{AB}$ and $\overrightarrow{DC}$ have opposite directions and are the same length then these are one of two pairs of opposite vectors. Another is $\overrightarrow{CA}$ and $\overrightarrow{BD}$.

Addition of vectors

The diagram below shows a point A being translated to point B then from that point B to a point C. A single displacement from A to C is the same as previously described.

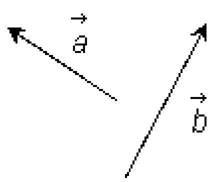
Three vectors together forming a triangle through addition is called the Triangle Law.



Triangle Law of Vector Addition
Let $\vec{a}$ and $\vec{b}$ be any two vectors arranged head-to-tail. The sum, $\vec{a}+\vec{b}$, is the vector from the tail of $\vec{a}$ to the head of $\vec{b}$.

Example 1

Below is given vectors $\vec{a}$ and $\vec{b}$.



a. Draw vector $\vec{a}+\vec{b}$.

b. Draw vector $\vec{b}+\vec{a}$.

c. Prove that $\vec{a}+\vec{b}=\vec{b}+\vec{a}$.

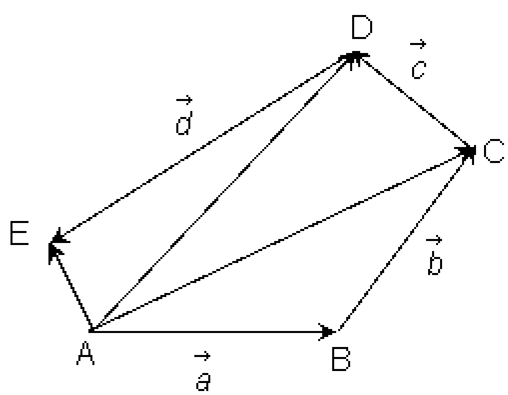
Adding more than two vectors together

To add three or more vectors, place them head to tail so that the tail of the second vector is at the head of the first, the tail of the third vector is a the head of the second vector, and so on.

The sum is the tail of the first vector and the head of the last vector.

Example 2

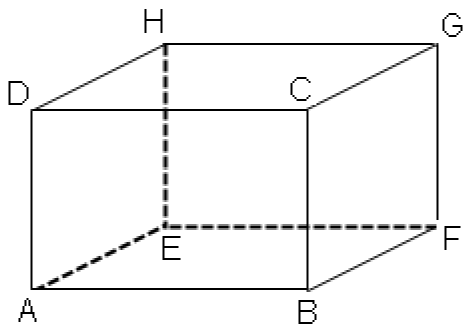
Find the sum of $\vec{a} + \vec{b} + \vec{c} + \vec{d}$



Example 3

The diagram below shows a rectangular prism. Determine a vector equal to each sum.

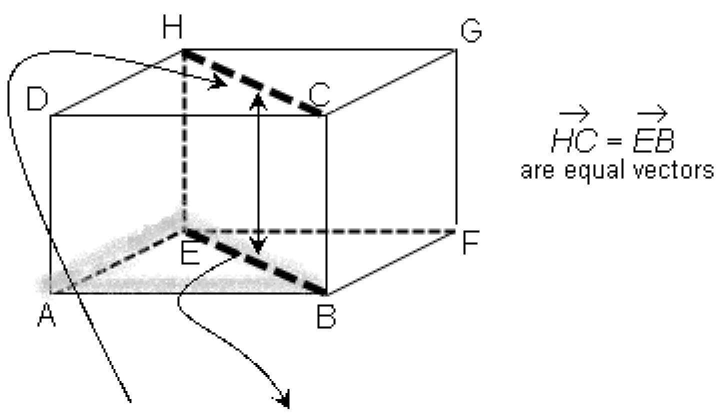
a. $\overrightarrow{AE} + \overrightarrow{HC}$ b. $\overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AB}$



Solution

a.

Place the vectors in order and add them head to tail. Where necessary, replace a vector with an equivalent vector to do the addition.



$$\begin{aligned}\overrightarrow{AE} + \overrightarrow{HC} &= \overrightarrow{AE} + \overrightarrow{EB}\\ &= \overrightarrow{AB}\end{aligned}$$

b.

$$\begin{aligned}\overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AB} &= \overrightarrow{AD} + \overrightarrow{DH} + \overrightarrow{HG}\\ &= \overrightarrow{AH} + \overrightarrow{HG}\\ &= \overrightarrow{AG}\end{aligned}$$

The Zero Vector

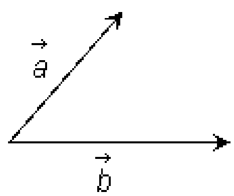
The zero vector has zero length and no specified direction. The sum of any vector and its opposite is the zero vector.

$\vec{a} + (-\vec{a}) = \vec{0}$

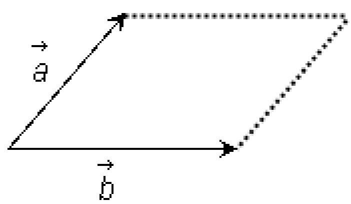
The zero vector, $\vec{0}$, is defined to be a vector so that the sum of any two vectors is always a vector. Hence, $\vec{0}$ is different than the number 0.

The Parallelogram Law of Vector Addition

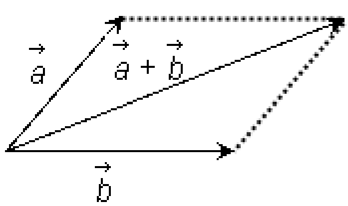
Let $\vec{a}$ and $\vec{b}$ be any two vectors arranged tail to tail.



Complete the parallelogram determined by $\vec{a}$ and $\vec{b}$.

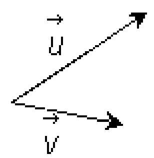


The sum of $\vec{a}$ and $\vec{b}$ is the vector with the same tail as $\vec{a}$ and $\vec{b}$ and with its head at the opposite vertex of the parallelogram.



Example 4

Draw $\vec{u}+\vec{v}$.



Solution